EDHEC-RISK INSTITUTE
EDHEC – 400 promenade des Anglais – 06000 Nice – France – Tel. +33 (0)4 93 18 66 99

Email: research@edhec-risk.com – Web: www.edhec-risk.com

Active Allocation to Smart Factor Indices

Noël Amenc, Guillaume Coqueret and Lionel Martellini

Abstract

While smart beta equity indices are often perceived as a threat for traditional active managers, this paper argues that they can also be regarded as an opportunity when used as efficient building blocks in active factor allocation strategies. We first show that substantial value can be added by time-varying *strategic* factor allocation decisions, where the focus is on efficiently reacting to changes in risk parameter estimates for various factor indices. In a second step, we show that additional value can be added by *tactical* factor allocation decisions based on an economic and statistical analysis of the conditional performance of smart factor indices for different types of market environment. Finally, we show that the strategically or tactically managed portfolio of smart factor indices can be used as an underlying satellite portfolio within a dynamic core-satellite strategy designed to generate a substantial access to the benefits of smart beta equity benchmarks with limited downside risk relative to the cap-weighted core portfolio.

1. Introduction: The Benefits of Multi Smart Beta Allocation

Once regarded as exotic curiosities, *smart beta equity indices* have now made it into the mainstream, and it has become of critical importance for investors and asset managers to better understand the sources of outperformance with respect to cap-weighted benchmarks, so as to better manage the associated risks. Smart equity indices in fact allow investors to benefit not from one but two distinct sources of added-value with respect to cap-weighted indices. The first source of added value comes from a more efficient exposure to rewarded risk factors. In particular, cap-weighted schemes inherently lead to a positive exposure to growth and large cap factor tilts, while an abundant academic research has shown that value and small cap tilts are instead rewarded in the long run (Fama and French (1992)). The second source of added value comes, for a given set of factor exposures, from a more efficient diversification of unrewarded risks, which are found in excessive amount in cap-weighted indices due to their excessive concentration.

New approaches to smart beta investing, and in particular the so-called smart beta 2.0 paradigm, precisely allow investors to frame their equity investments within a coherent framework that is consistent with the principles of modern portfolio theory. Asset pricing theory shows that individual stocks earn their risk premium from exposure to rewarded risk factors (see Merton (1973) or Ross (1976) for equilibrium or arbitrage arguments, respectively). When systematic biases in factor exposure are explicitly controlled for, smart beta investing allows investors to effectively harvest the risk premium on these factors through an efficient diversification of stocks' specific risks (see in particular Amenc et al. (2012, 2013, 2014)). In this context, investors in smart betas will benefit from allocating to multiple systematic factor exposures so as to generate optimal risk-adjusted performance (Cochrane (1999)).

The main focus of this research project is to provide a formal empirical analysis of the benefits of strategic and tactical allocation to multiple equity smart factor indices in a context where relative risk with respect to the cap-weighted indices needs to be explicitly controlled for. As such, our paper extends existing research on regime-based tactical allocation strategies (e.g. Chow et al. (1999), Bauer et al. (2004) or Sheikh and Sun (2012)) that has mostly focused on allocation shifts between stocks, bonds and commodities. Broadly speaking there are 3 mains sources of added-value in time-varying allocation decisions to smart factor indices. The first source of added value, analyzed in Section 2, comes from time-varying *strategic* allocation decisions, where the focus is on efficiently reacting to changes in risk parameter estimates. The second source of added value, analyzed in Section 3, comes from time-varying *tactical* allocation decisions, where the focus is on efficiently reacting to changes in market conditions based on a detailed analysis of the conditional performance of smart factor indices for different types of market environment. The third and last source of added-value, analyzed in Section 4, comes from time-varying *core-satellite* allocation decisions, where the focus is on efficiently managing the portfolio risk with respect to the cap-weighted reference, so as to generate a substantial access to the benefits of smart beta management, with limited downside risk relative to the cap-weighted benchmark. The focus of this paper is to provide a quantitative assessment of the benefits expected from these three sources of added-value in the design of equity benchmark with superior risk and return characteristics. Finally, our conclusions and suggestions for further research can be found in Section 5.

2. Designing a Strategic Policy Portfolio of Smart Equity Factor Indices

The first dimension to account for in the context of smart beta investing is the design of a benchmark which aims to represent the strategic set of exposures to rewarded risk factors that are sought by the investor. This benchmark contains the main sources of risk-adjusted performance, which can be expressed either from an absolute or from a relative perspective with respect to a cap-weighted index.

2.1. Selecting Smart Factor Index Building Blocks

In this Section, we review the rationale for the use of smart factor indices. We first document the two main shortcomings of cap-weighted indices, namely their inefficient exposures to rewarded factor tilts, and their excessive concentration which results in an excessive amount of unrewarded risks.

2.1.1. Shortcomings of Cap-Weighted Indices

Asset pricing theory suggests that there are two main challenges involved in a sound approach to equity investing. The first challenge is the efficient diversification of unrewarded risks, where "diversification" means "reduction" or "cancellation" (as in "diversify away"). Indeed, unrewarded risks are by definition not attractive for investors who are inherently risk-averse and therefore only willing to take risks if there is an associated reward to be expected in exchange for such risk taking. The second challenge is the efficient diversification of rewarded risks. Here the goal is not to diversify away rewarded risk exposures so as to eventually eliminate or at least minimize them, since this would imply giving up on the risk premia; the goal is instead to efficiently allocate to rewarded risk factors so as to achieve the highest reward per unit of risk. This analysis of the dual challenges to rational equity investing is enlightening with respect to a proper understanding of the intrinsic shortcomings of cap-weighted (CW) indices that are typically used as default investment benchmarks by asset owners and asset managers. On the one hand, CW indices are ill-suited investment benchmarks because they tend to be concentrated portfolios that contain an excessive amount of unrewarded risk. On the other hand, CW indices implicitly embed a bundle of factor exposures that are highly unlikely to be optimal for any investor, if only because they have not been explicitly controlled for. For example, CW indices show by construction a large cap bias and a growth bias, while the academic literature has instead shown that small cap and value where the positively rewarded risk exposures. We conclude this Section by some empirical evidence of the inefficient systematic and specific risk exposures in cap-weighted indices.

2.1.2. Introducing Smart Factor Indices

Asset pricing theory suggests that security and asset class returns can be explained by their exposure to pervasive systematic risk factors**.** More precisely, asset pricing models relate expected returns on financial assets to the co-movements between future realized returns and a stochastic discount factor, also known as pricing kernel, which is related to marginal utility of consumption (see for example reference textbooks by Duffie (2001) and Cochrane (2005)).[1] Let us define the "bad" states of the world as those where consumption is low, and marginal utility of consumption is high. Asset pricing theory

[1] The existence of the stochastic discount factor can also be justified by the absence of arbitrage.

states that an investor is ready to accept a lower expected return, i.e., pay a higher price, to hold an asset that pays off well in those bad states. Conversely, an asset that tends to pay off well when marginal utility of consumption is low is less appealing to investors, who thus require a higher expected return, i.e., a lower price, to purchase it.

While the consumption-based asset pricing models allow us to relate the stochastic discount factor to aggregate consumption, such models can hardly be used in practice because macroeconomic aggregate consumption measures are only available at a relatively low frequency, are released with a lag and are subject to ex-post revisions. In this context, one natural approach consists of searching for other, more easily observable, factors that can be used as proxies for the unobservable stochastic discount factor. The first examples of multi-factor asset pricing models can be traced back to the intertemporal CAPM (Merton (1973)) or the arbitrage pricing theory (Ross (1976)), following ion the development of the static single-factor CAPM model by Sharpe (1964). This well-understood paradigm has had extremely important implications for evaluating the performance of active managers. Thanks to the operationalization of multi-factor models by various software/data providers, asset owners, (multi) managers and consultants have developed the ability to generate more precise performance measurements for active funds.

The idea here is to assess what an active manager normal performance should be as a by product of the manager's factor exposures, which in turn allows one to assess what the active value-added, also known as alpha, truly is. In an influential empirical analysis of the performance of the Norwegian Government Pension Fund Ang, Goetzmann and Schaefer (2009)) have found that the value-added by active managers was extremely limited above and beyond the fair reward to be expected from their risk exposures. These findings suggest that an efficient approach to equity investment should focus on harvesting risk premia through allocation decisions to well-rewarded risk factors.

In the subsequent Section, following this profound paradigm shift away from active management and towards efficient beta management, we provide a detailed overview of factor investing in the equity space. We first present a list of rewarded factor tilts that have been unveiled by more than 40 years of academic and practitioner research on equity markets. We make a distinction between factor premia that have a rational versus behavioral explanation, and also discuss factor premia for which there is no well-understood rational or behavioral explanation, and which may be regarded as anomalies. We subsequently focus on four well-known rewarded factors, namely the Size and Value factors (Fama and French (1992)), the Momentum factor (Carhart (1997)) and the Low Volatility factor (Ang et al. (2006, 2009)).

For each rewarded factor, we introduce a corresponding smart factor index, which can be regarded as an efficient investable proxy for a given risk premium. In a nutshell, a risk premium can be thought of as a combination of a risk (exposure) and a premium (to be earned from the risk exposure). Smart factor indices have been precisely engineered to achieve a pronounced factor tilt emanating from the stock selection procedure (relevant risk exposure), as well as high Sharpe ratio emanating from the efficient diversification of unrewarded risks related to individual stocks (fair reward for the risk exposure).

The access to the fair reward for the given risk exposure is obtained through a well-diversified, also known as smart-weighted, portfolio, as opposed to a concentrated cap-weighted portfolio, of the selected stocks so as to ensure that the largest possible fraction of individual stocks' unrewarded risks is eliminated. The empirical analysis will be performed on long-term US data (over the 1972-2013 period).

To illustrate the benefits of smart factor indices, we provide common performance indicators in Table 1 . The benchmark is the broad cap-weighted (CW) index (in this case the S&P500 index). Next, we have split the universe in two and only kept the stocks with the rewarded characteristic (low market capitalization, high book-to-market, high past annual return or low realized volatility in the past two years). For all of these tilts, we tested two weighted schemes: proportional to capitalization (CW) or uniform across all stocks (EW). The universe is updated every year but a rebalancing occurs quarterly in order to track changes in the S&P500 composition.

Table 1 : Performance of smart factor indices

Selection	All	Mid Cap		Value		Momentum		Low Volatility	
Weighting	CW	CW	EW	CW	EW	CW	EW	CW	EW
Ann. Return	10.5%	13.0%	14.0%	12.4%	14.7%	11.6%	13.9%	10.8%	13.1%
Ann. Volatility	15.7%	18.4%	19.8%	16.3%	17.9%	15.8%	16.8%	14.1%	15.0%
Sharpe Ratio	0.33	0.41	0.43	0.43	0.52	0.40	0.51	0.38	0.52
Max. Drawdown	52.4%	58.0%	60.1%	59.2%	60.3%	45.6%	49.9%	48.2%	52.1%
Excess Return vs CW	-	2.5%	3.5%	1.9%	4.2%	1.1%	3.4%	0.3%	2.6%
Tracking Error vs CW	-	7.2%	9.0%	5.1%	7.4%	4.1%	5.1%	4.7%	6.1%
Information Ratio vs CW	-	0.35	0.39	0.37	0.57	0.27	0.66	0.06	0.43
Rel. Max. Drawdown	-	35.0%	41.6%	19.1%	30.8%	13.2%	14.1%	31.4%	39.9%

The indices originate from the Scientific Beta platform. The sample starts in January 1972 and ends in December 2013. The first four indicators are the following: the geometric annualized return, the standard deviation of monthly returns multiplied by $\sqrt{12}$, the annualized return of the portfolio minus that of the 3M US Treasury Bill divided by the annualized volatility and the maximum drawdown of the values of the portfolio. The last four indicators measure performance relative to that of the broad CW index: the excess annualized return, the volatility of excess returns, the ratio of the former by the latter (IR) and lastly, the maximum drawdown of the ratio of the values (portfolios divided by CW).

We first confirm that over our sample period, the selected factor tilts are indeed rewarded. With a similar cap-weighted scheme, the improvement in return lies between 30 basis points (low volatility) to 250 basis points (mid capitalization). The increase in Sharpe ratio ranges from +0.05 (low volatility) to +0.08 (value) in absolute value. The extreme risk indicator (maximum drawdown) is not homogeneous across the factors: from 48% for the low volatility tilt, it goes up to 59% for the value factor. In terms of relative drawdown however, the figures reverse and the value tilt (19%) is much more competitive than the low volatility tilt.

Second, we find that these gains are further enhanced when switching from a CW to an EW weighting scheme, which implies the lowest degree of concentration and hence a better diversification at least from a naïve standpoint. For a given tilt, the increase in returns ranges from +1% (mid cap) to +2.4% (low volatility). In terms of Sharpe ratio, the improvement is +0.02 for the mid-cap indices and +0.14 for the low volatility indices. However, the maximum drawdown is usually higher when switching to EW factors, but the impact is rather limited (+4% at most). The relative performance is also improved by the smart weighting: while the information ratio gains only +0.04 in absolute value for the mid-cap factors, the increase reaches +0.37 for the low volatility indices.

In sum, smart factors do improve classic passive strategies both by selecting stocks that are, on average, more rewarded in the cross-section, and by allocating to these stocks in a diversified manner that allows for enhanced diversification benefits (see Amenc et al. (2014) for more details).

2.2. Allocating to Smart Factor Index Building Blocks

In this Section, we seek to build a strategic benchmark, based on reliable risk estimates only, as opposed to forward-looking noisy expected return estimates. In this context, risk budgeting techniques appear to be naturally suited for constructing a well-diversified benchmark of smart beta strategies. Risk parity has indeed become an increasingly popular approach for portfolio construction within and across asset classes. In a nutshell, the goal of the methodology is to ensure that the contribution to the overall risk of the portfolio will be identical for all constituent assets, which stands in contrast to an equally-weighted strategy that would recommend an equal contribution in terms of dollar budgets. More generally, one can envision to assign equal or non-equal target budgets for the contribution of each constituent to the risk of the overall portfolio. In the specific case of an equal contribution, risk parity is sometimes also known as equal risk contribution (ERC).

In what follows, we therefore introduce strategic factor allocation strategies relying on risk budgeting portfolio techniques and on smart factor indices as well diversified and factor controlled building blocks. An in-depth overview of the benefits and limits of risk parity portfolios, and more generally to risk budgeting portfolios which assign non necessarily equal risk budgets for constituents, is given in Roncalli (2013), who shows that risk parity is a disciplined approach to portfolio diversification that is more meaningful than naive approaches based on dollar budgets, and more robust with respect to errors in input parameter estimates compared to standard mean-variance analysis.

2.2.1. Risk Parity/Budgeting as an Objective

Risk parity is a specific case of risk budgeting, and is consistent, when applied to uncorrelated factors, with Sharpe ratio optimization assuming constant Sharpe ratios (Deguest, Martellini and Meucci (2013)). We apply the methodology of Maillard et al. (2010) to the design of a strategic portfolio of smart factor indices. More precisely, while a naive equally-weighted allocation would invest $w_i = 1/N$ in each index (N being the number of indices), risk parity weights are computed so as to equalize the contribution of assets to portfolio risk. The volatility of the aggregated portfolio of indices is first decomposed into:

$$\sigma = \sum_{i=1}^{N} RC_i, \quad \text{where the risk contributions are equal to } RC_i = w_i \frac{[\boldsymbol{\Sigma w}]_i}{\sqrt{\boldsymbol{w}'\boldsymbol{\Sigma w}}},$$

and risk parity is achieved when $RC_i = 1/N$ for all indices. As such, the major difference between the two approaches is that 1/N weights imply a homogeneous allocation in terms of dollars, whereas risk parity refers to a uniform contribution of the indices to the *volatility* of the portfolio.

We provide below (in Table 2) the performance indicator of portfolio based on the risk parity approach.

Table 2 : Performance of heuristic or risk-driven allocations to smart factors

	CW (All stocks)	EW (EW)	EW (CW)	ERC(EW)	ERC (CW)
Ann Return	10.5%	14.0%	12.0%	13.9%	12.0%
Ann Volatility	15.7%	17.0%	15.6%	16.8%	15.4%
Sharpe Ratio	0.33	0.51	0.43	0.51	0.43
Max Drawdown	52.4%	55.7%	52.6%	55.4%	52.00%
Excess Return vs CW	-	3.5%	1.6%	3.4%	1.5%
Tracking Error vs CW	-	5.9%	2.9%	5.8%	3.0%
Information Ratio	-	0.59	0.53	0.59	0.51

This table displays the performance indicators for portfolios which allocate across four smart factors. The first column relates to the broad CW benchmark. In the following two columns, the allocation is uniform across assets and in the last two columns, the allocation is performed so as to equate the risk contributions of the factors (ERC). Within the factors, the stocks are either weighted proportionally to capitalization (CW) or uniformly (EW=equally weighted). The first four indicators are the following: the geometric annualized return, the standard deviation of monthly returns multiplied by $\sqrt{12}$, the annualized return of the portfolio minus that of the 3M US Treasury Bill divided by the annualized volatility and the maximum drawdown of the values of the portfolio. The last three indicators measure performance relative to that of the broad CW index: the excess annualized return, the volatility of excess returns, the ratio of the former by the latter (IR).

When switching from a uniform to a ERC allocation across factors, the results are only slightly impacted. This comes from the fact that both correlations among factors and volatilities across factors are rather homogeneous, and hence, the weights implied from the ERC optimization are not too far from evenly distributed. Consequently, EW and ERC yield close results. We confirm, in passing, that the allocations which are based on maximum diversification blocks (i.e. close to equally weighted) generate higher returns and Sharpe ratios, compared to their cap-weighted counterparts. Quite naturally, the CW indices have a much smaller tracking error but the net effect on the information ratio is to the advantage of the EW factors which further highlights their superiority in the satisfaction of investor welfare.

2.2.2. From Absolute to Relative Risk Control

It is often the case that investors maintain the cap-weighted index as a benchmark, which has the merit of macro-consistency and is well-understood by all stakeholders. Macro-consistency implies that a cap-weighting index is by definition the only index that can be held by *all* investors. On the other hand, it of course does not imply that some investors cannot and should not invest in non cap-weighted indices. Any index with an alternative smart weighting scheme can perfectly be held by some investors, as long as there remains a set of investors who are prepared to hold on aggregate a portfolio that offsets the tilts of the smart-weighted index with respect to the cap-weighted index. In practice, investability of smart-weighted indices is therefore not a concern especially in liquid large and mid-cap developed markets. In a context where the cap-weighted index remains the ultimate reference, a multi smart beta solution can be regarded as a reliable cost-efficient substitute to expensive active managers, and the most relevant perspective is not an absolute return perspective but a perspective relative with respect to the cap-weighted index. In this subsection, we introduce a robust approach leading to a relative equal risk allocation (R-ERC) portfolio, which focuses on equalizing the contribution of the smart factor-tilted indices to the portfolio tracking error. This portfolio will be used as a strategic benchmark in the empirical exercises from the next two Sections. A similar analysis could be performed, with qualitatively similar results, based on the use of a different strategic benchmark. In particular, an active manager with loose or no tracking error constraints with respect to the cap-weighted index would construct a strategic benchmark with no explicit reference to the cap-weighted index.

More formally, the R-ERC approach can be formulated as follows. Similarly to the base-case ERC, the optimal weights are defined implicitly by

$$RC_i = w_i \frac{[\widetilde{\mathbf{\Sigma}}\boldsymbol{w}]_i}{\sqrt{\boldsymbol{w}'\widetilde{\mathbf{\Sigma}}\boldsymbol{w}}} = \frac{1}{N}, \quad \forall i = 1, \dots, N,$$

but in this case, the covariance matrix $\widetilde{\mathbf{\Sigma}}$ is not that of the raw returns of the factors, but that of the returns net of the returns of the CW benchmark. Hence, the optimal weights will equate the contribution of the factors to the *tracking error* with respect to the benchmark (and not to the volatility of the portfolio).

The corresponding results (allocation to the four tilted factors) are gathered in Table 3.

Table 3 : Performance of heuristic or relative risk-driven allocations to smart factors

	CW (All stocks)	EW (EW)	EW (CW)	R-ERC(EW)	R-ERC (CW)
Ann Return	10.5%	14.0%	12.0%	13.7%	11.4%
Ann Volatility	15.7%	17.0%	15.6%	16.5%	15.3%
Sharpe Ratio	0.33	0.51	0.43	0.51	0.40
Max Drawdown	52.4%	55.7%	52.6%	54.7%	51.6%
Excess Return vs CW	-	3.5%	1.6%	3.2%	1.0%
Tracking Error vs CW	-	5.9%	2.9%	5.0%	1.9%
Information Ratio	-	0.59	0.53	0.64	0.49

This table displays the performance indicators for portfolios which allocate across four smart factors. The first column relates to the broad CW benchmark. In the following two columns, the allocation is uniform across assets and in the last two columns, the allocation is performed so as to equate the risk contributions of the factors (R-ERC), relatively to the CW benchmark returns. Within the factors, the stocks are either weighted proportionally to capitalization (CW) or uniformly (EW=equally weighted). The first four indicators are the following: the geometric annualized return, the standard deviation of monthly returns multiplied by $\sqrt{12}$, the annualized return of the portfolio minus that of the 3M US Treasury Bill divided by the annualized volatility and the maximum drawdown of the values of the portfolio. The last three indicators measure performance relative to that of the broad CW index: the excess annualized return, the volatility of excess returns, the ratio of the former by the latter (IR).

Compared to the previous table, we see that the difference between a uniform allocation and the R-ERC weighting is more pronounced. Both the returns and the volatility are reduced and the net effect is that the Sharpe ratio remains close to unchanged. The maximum drawdown is also slightly reduced when using the R-ERC scheme. In terms of relative performance, the R-ERC strongly reduces the tracking error with respect to the CW benchmark. The impact on the information ratio depends on the performance of the underlying factors. If the factors strongly outperform the benchmark (EW), then the R-ERC improves the IR, if not (CW), then it is the opposite.

Given these promising results, we will consider in the subsequent sections two alternative benchmarks to the CW index: the equally weighted portfolio of the four smart factors and the corresponding portfolio based on the R-ERC allocation.

3. Outperforming the Strategic Policy Portfolio Strategy with Tactical Tilts

Current research suggests that the distribution of asset returns is time-varying, characterized by periods of turbulence with high volatility and low returns followed by calmer periods with low volatility and above average returns. In particular, in the high volatility regime equity market returns are negative while the Sharpe ratio for the value, small cap and momentum premia is close to zero, and the low vol premium becomes particularly attractive.

In the presence of regimes, portfolio managers should hold different portfolios depending on their forecasts of the future risk state. In particular, they should scale down the volatility of their portfolios when volatility is high and increase the risk of their portfolios when volatility is below average. Regime based active portfolio management is increasingly perceived as a robust approach to generating outperformance with respect to a static strategic benchmark in international markets (Angelidis and Tessaromatis (2014a, 2014b)).

Given that smart beta indices appear as the natural building blocks to use in an active allocation strategy, the focus of this Section is precisely to analyze whether superior performance can be achieved by over/under-weighting smart factor indices with respect to the strategic benchmark from Section 2 as a function of changes in market conditions such as formally represented by well-chosen regimes. So as to avoid uncontrolled deviations from the strategic allocation benchmark, we shall only use smart factor indices that are chosen in the same list as those composing the strategic benchmark. Note that we have a minimum allocation to zero, with no short position allowed.

Our work will build on prior research on the benefits of style rotation strategies (growth, value, small cap, etc.) as a function of regimes that can be defined from a purely statistical sense (hidden Markov models) or preferably from explicit state variable (regime switching models based on well-chosen state variables - see the discussion in Section 3.1.1 below). This work will be extended to encompass smart weighted constituents (as opposed to cap-weighted constituents), and also to encompass a set of rewarded factors such as momentum and low vol, which have not been traditionally perceived as investment styles.

3.1. Construction of a Parsimonious and Robust Regime Switching Model

Since the seminal work by Hamilton (1989), applications of regime switching models have blossomed in finance. They include interest rates modeling (Ang and Bekaert (2002a)), equity return predictability (Henkel et al. (2011)) and asset allocation decisions (Ang and Bekaert (2002b)). We refer to Ang and Timmermann (2012) for an exhaustive survey on this topic.

3.1.1. Regime Switching Models based on Observable versus Latent State Variables

In this Section, we review the empirical evidence for Hidden Markov regime switching models versus regime switching models based on observable variables.

Originally, Markov switching regimes were developed to detect changes in the behavior of time-series. For instance, they can be used to determine business cycles in economics or periods of turbulence in stock markets. However, a robust estimation of the parameters of the model requires large sample sizes because regimes can only be identified in the long term. Once the model is estimated, it is usually able to describe the regimes within the original sample. As a result, in-sample predictability or asset allocation exercises produce impressive results. However, real-time out-of-sample performance is typically much weaker due to the presence of parameter instability (Marsh (2000)), which may lead to misclassification of regime shifts and hence erroneous decisions. Consequently, the benefits of Markov switching models compared to simpler approaches are strongly reduced in out-of-sample asset allocation exercises, especially when shortsales are not allowed (e.g., Ammann and Verhofen (2006), Chang (2009), Tu (2010), Angelidis and Tessaromatis (2014a)).

In this context, and given the intrinsic difficulty in the robust calibration of Markov regime switching models from an out-of-sample perspective, we use in what follows regime switching models based on observable variables.

3.1.2. Documented Evidence of Time-Varying Factor Performance

Since our ultimate goal is to dynamically allocate across factor indices on the basis of observable factors, we must first identify the state variables which drive the performance of each factors.

3.1.2.1. The Size Factor

The size premium, namely the documented finding that firms with smaller capitalization tend to outperform firms with large capitalization in the long run, was initially unveiled by Banz (1981) and Reinganum (1981). Subsequent studies (Brown et al. (1983), Keim (1983), Lamoureux and Sanger (1989) and Fama and French (1992)) confirmed their original findings. Even though some authors have questioned the persistence of the phenomenon (Dimson and Marsh (1999), Schwert (2003)), van Dijk (2011) in his survey of the size premium concludes that "the size premium in the US has been large and positive in the recent years".

Obviously that the returns on small cap firms dominates the return on large cap firms on average across all market conditions does not imply that this outperformance of small cap stocks is expected to be constant for any market condition and a proper understanding the drivers of these fluctuations is critical for the purpose of the current study (see for example Jagannathan and Wang (1996), among others, for a conditional analysis of the time variations of the risk premia associated to portfolios sorted on size). We summarize below the macro-economic and financial variables which are expected to impact the performance of long-short portfolios based on size.

First of all, economic cycles are likely to impact the performance of the size factor. Small firms are more vulnerable in recession states. Burnie and Kim (2002) show that small firms perform better than large firms in expansion states, but that the size premium vanishes in the recession state. Moreover, because of their higher flexibility, small firms outperform large firms in the recovery period after a recession (Switzer (2010)). Empirically, these effects should be captured using NBER cycles or economic growth and inflation indicators, but the term spread is another relevant indicator. Indeed, as shown by Estrella and Mishkin (1998), the slope of the yield curve is the most important determinant of future economic growth (an inverted yield curve signals a recession). We therefore expect the term spread to be positively correlated with the size premium (see also Petkova (2006)).

Another set of macroeconomic variables related to credit conditions. For instance, higher interest rates make it difficult for small firms to finance projects typically because rising interest rates weaken firms' balances sheets by lowering the present value of their collateral (Kiyotaki and Moore (1997)). High (or increasing) interest rates should thus be associated with negative returns for long-short portfolios based on size. In a similar vein, liquidity is a risk factor (Acharya and Pedersen (2005)) and low liquidity is expected to impact small firms harder than large firms. Pastor and Stambaugh (2003) find that portfolios of small firms have the highest loadings on their liquidity factor. Amihud (2002) also documents that returns of small firms are sensitive to variations in market liquidity. Credit conditions are also often straightforwardly associated to the credit spread and Fama and French (1993) show that small firms are significantly more exposed to the default spread than large firms. Hahn and Lee (2006) report a negative exposure of the size factor to *variations* in the default spread. The rationale behind these results is that small firms are more vulnerable (because of smaller collateral) to worsening credit conditions. This is consistent with the findings of Vassalou and Xing (2004) who show that, in the cross-section, default risk is a decreasing function of firm size.

The conditional behavior of the size premium is also impacted by financial variables. Since small stocks usually have larger market betas than large stocks, the SMB portfolio has a positive beta and covaries positively with market returns. As reported by Arshanapalli et al. (2006), Arshanapalli and Nelson (2007) and Hur et al. (2014), small firm returns are higher than large firms returns when the market rises, but the opposite occurs in times of market decline. Past market returns also matter: Lo and MacKinlay (1990) find that returns on large stocks lead those on smaller stocks. This lead-lag effect implies that market returns (mainly driven by large stocks) are positively correlated to future small cap stock returns. Lo and MacKinlay find a positive correlation for short horizon returns (week or months), but for longer horizon the effect might reverse because prolonged negative returns will be followed by positive market returns which are favorable to small stocks. Aggregate risk is also likely to play a key role for the size factor. Indeed, investors tend to fly to quality when their expectation of risk increases (Durand et al. (2011)). We thus expect small firms to have lower returns in times of high volatility (Copeland and Copeland (1999) show that on days that follow increases in the VIX, portfolios of large-capitalization stocks outperform portfolios of small-capitalization stocks).

Lastly, factor-specific variables, and in particular the factor past performance, may also influence the performance of the factor (see the seminal article of Barberis and Shleifer (2003) for an analysis of style momentum). Wang (2003), Chen and De Bondt (2004) and Teo and Woo (2004) document a phenomenon of style momentum: strategies that buy stocks with characteristics that are currently in favor (past winners) and that sell stocks with characteristics that are out-of-favor (past losers) perform well for periods up to 1 year and possibly longer. As such, the past performance of the factor is a driver of its future payoffs. This effect has also been identified in global markets (Aarts and Lehnert (2005), Chao et al. (2012)), and applies to the size factor just as it applies in principle to all factors analyzed in our paper.

3.1.2.2. *The Value Factor*

The foundation of value investing dates back to Graham and Dodd (1934) who argue that securities should be purchased if their market prices are less than their intrinsic values. The strategy which consists in buying stocks with high book-to-market ratio and selling those with low book-to-market ratio has yielded large positive returns in the long run in the US market (see for example Rosenberg et al. (1985), Fama and French (1992) and Lettau and Wachter (2007), among many others). The value premium has also been documented in international markets (Fama and French (1998) and Asness et al. (2013)).

Two explanations have been provided to explain the value premium. The first explanation is a rational risk-based explanation: value stocks are more rewarded in the long run because they are riskier in bad times. Petkova and Zhang (2005) and Choi (2013) for instance provide empirical support for this conditional behavior. Theoretical models have also been proposed that confirm the counter-cyclicality of portfolios sorted on book-to-market. Zhang (2005) and Cooper (2006) propose models based on costly reversibility of capital, i.e., the fact that firms face higher costs in cutting than in expanding capital. However, many authors have put forward empirical evidence against the risk-based explanation of the value premium (Chan and Lakonishok (2004), Arshanapalli et al. (2006), Arshanapalli and Nelson (2007), Phalippou (2007), Cooper and Gubellini (2011), Ang and Kristensen (2012), Fong (2012)).

Lakonishok et al. (1994), Laporta (1996) and Laporta et al. (1997) provide a competing behavioral explanation. Investors' decisions are plagued by judgment biases and forecast errors as a result of which they underprice out-of-favour (value) stocks and overprice glamour (growth) stocks. Hwang and Rubesam (2013) argue that value stocks outperform growth during bad times because the underpricing of value stocks is corrected faster during bear market when volatility increases.

Similarly to the size factor, broad economic conditions are expected to alter the profits to the value strategy. According to Hwang and Rubesam (2013), the underpricing of value stocks is expected to be corrected faster during recessions. Lakonishok et al. (1994) for instance find that on average, value stocks perform better than growth stocks during NBER recessions. Interest rates may also impact the value premium: one common characteristic of most growth stocks is a lower propensity to pay dividend to shareholders, while value stocks tend to pay out much more of their net income in the form of a current dividend. Therefore growth stocks can be said to have a longer "duration" than value stocks (Lettau and Wachter (2007)). Thus, we would expect growth stocks to underperform in an environment of steeper yield curves, which imply expectations of rising interest rates in the future. To confirm this intuition, Amenc et al. (2003) on a sample from September 1991 to May 2002 find that flattening moves for the yield curve are associated with an outperformance of growth stocks. Lastly, other macroeconomic variables related with documented influence on the value strategy include the default (i.e., credit) spread (in US markets) and liquidity indicators (Asness et al. (2013)).

With regard to financial variables, there is abundant empirical evidence that the value premium is countercyclical. Chan and Lakonishok (2004), Arshanapalli et al. (2006), Arshanapalli and Nelson (2007) all document that returns of value stocks are higher in times of market downturn. Hwang and Rubesam (2013) argue that value stocks outperform growth during bad times because the underpricing of value stocks is corrected faster during bear market when volatility is high. Value payoffs are therefore expected to be negatively related to current market returns. The findings of Hwang and Rubesam (2013) also suggest that the value premium is positively correlated with volatility. Moreover, Copeland and Copeland (1999) show that value stocks outperform growth stocks when the VIX index increases. In a similar vein, Lee and Song (2003) report that the value premium is higher when the VIX index is above its past 6 month average. Another related variable is the cross-sectional dispersion and Chen and Stivers (2010) find significant positive relationship between this variable and returns of the value strategy.

Finally, as for the size factor, the value factor is expected to be driven by its past performance (style momentum), but also by its current volatility. Indeed, using a generalization of the GARCH model, Li et al. (2009) show that the value premium is significantly and positively correlated with its volatility.

3.1.2.3. *The Momentum Factor*

A momentum strategy consists of buying stock with the highest performance over the past year and selling stocks with the lowest performance over the same period of time. The sustainable positive returns associated to this strategy were first documented in the US by Jegadeesh and Titman (1993) and subsequently confirmed for global markets by Rouwenhorst (1998), Griffin, Ji, and Martin (2003) and Chui, Titman and Wei (2010).

Most of the explanations behind the momentum effect are based upon the existence of positive serial correlation of individual stock returns, which is broadly consistent with empirical evidence. The theories differ as to whether the serial correlation is caused by under-reaction (Barberis, Shleifer and Vishny (1998)) or delayed overreaction (Delong, Shleifer, Summers and Waldman (1990)). Another stream of the literature follows the seminal “self-attribution bias” explanation introduced by Daniel, Hirshleifer and Subramanyam (1998), whereby informed traders attribute the performance of ex-post winners to their stock selection skills and that of the ex-post losers to bad luck. These investors become overconfident about their ability to pick winners and overestimate the precision of their signals for these stocks. Based on their increased confidence in their signals, they push up the prices of the winners above their fundamental values.

We now turn to an analysis of the economic and financial variables which are likely to impact the momentum strategy. Asness et al. (2013) argue that when a liquidity shock occurs, investors engaged in liquidating sell-offs (due to cash needs and risk management) will put more price pressure on the most popular and crowded trades, such as high momentum securities, as everyone runs for the exit at the same time. Accordingly, momentum returns are expected to be low in times of low liquidity. Avramov et al. (2014) confirm this relationship and further underline that all behavioral explanations for the momentum premium hold all the more so when liquidity is high. Consequently, we expect momentum returns to be positively related to liquidity or liquidity shocks.

Market conditions also drive the performance of the momentum strategy. Daniel and Moskowitz (2013) and Barroso and Santa-Clara (2015) document the so-called “*momentum crash*” effect. In “panic” states, following negative returns for the overall market, winners tend to be low-beta stocks and the reverse for losers. Therefore, the winner-minus-losers strategy has a negative beta in the midst of market rebounds.

As such, following negative market returns and high volatility, we expect negative payoffs for the momentum strategy. Each variable can also be taken separately: Cooper et al. (2004) show that momentum returns are positively related with *past* returns on the market, and Wang and Xu (2015) find that momentum profits are negatively impacted by past market volatility. In a similar vein, Stivers and Sun (2010) report that cross-sectional dispersion is empirically negatively related to momentum returns.

3.1.2.4. *The Volatility Factor*

Haugen and Baker (1991) were the first to show that global minimum variance benchmarks that overweight in low volatility stocks were an efficient alternative to standard cap-weighted indices. More recently, Ang et al. (2006), Baker et al. (2011) and Frazzini and Pedersen (2014) have shown that stocks with low idiosyncratic risk, low volatility or low beta outperformed their risky counterparts in the long run. This phenomenon is persistent in global markets, as was shown by Ang et al. (2009), Guo and Savickas (2010) and Baker and Haugen (2012).

Several arguments have been put forward to explain this “*low volatility*” anomaly such as the presence of fixed benchmark mandates (Baker et al. (2011)), irrationality of investors who seek lottery tickets (Mitton and Vorkink (2007), Ilmanen (2012)) or leverage constraints (Frazzini and Pedersen (2014)). Lastly, the *variance drain effect* (Becker (2012)) also straightforwardly explains why geometric returns are negatively impacted by larger dispersions in arithmetic returns.

We now turn to an analysis of the variables that are likely to influence the performance of the volatility strategy. First of all, low volatility stocks are more correlated to bonds than their high volatility counterparts (Baker and Wurgler (2012) and Coqueret et al. (2014)). Accordingly, the long-short portfolio based on volatility is expected to be negatively impacted by rising interest rates. Also, market states are likely to alter the performance of strategies based on cross-sectional volatilities: since the low-volatility stocks have market betas which are lower than their high-volatility peers, the market beta of the volatility factor (long low volatility and short high volatility) is negative. As such, the returns on the long-short volatility strategy are expected to be negatively correlated with market returns. Furthermore, high volatility stocks have a higher exposure to volatility than low volatility stocks so the volatility factor has a negative exposure to aggregate volatility. Hence, the volatility strategy should have lower returns when volatility is high. This has been confirmed by Ang et al. (2006) when the stability of regimes is assessed through absolute value of monthly returns.

3.1.3. Investor's Sentiment Variables

In addition to macro-economic and financial variables that are likely to provide conditional information about future cash-flows expected from holding risky securities, asset pricing theory suggests that investor's risk aversion or risk appetite is an additional ingredient that has a strong impact on the prices and returns on these securities. The Asian financial crisis of 1997, the aftermath of the Russian debt default of 1998, and the collapse of high-technology stock prices in 2000 are a few examples of events that appear to be related to systemic changes in investors' appetite for risk.

Not surprisingly, a growing number of financial institutions and organizations have been developing measures of risk appetite in an effort to quantify this phenomenon. These include the JP Morgan Liquidity, Credit, and Volatility Index (LCVI), the UBS Investor Sentiment Index (UBS), the Merrill Lynch Financial Stress Index (ML), and the Westpac Risk Appetite Index (WP), among others. Other related indicators include the Credit Suisse Risk Appetite Investable Index, which results from a rule-based asset allocation methodology between stocks and bonds designed to outperform a neutral 50%-50% stock-bond benchmark, or the VIX index, also known as the "fear index", which is a measure of option-implied equity volatility put together by the CBOE, and which is of course highly related to the measure of historical volatility discussed in the previous section.

We provide below an empirical analysis of these risk indicators and their relation with respect to afore-mentioned financial variables, in an attempt to see whether they could be usefully included in the list of state variables used to define market regimes.

In detail, we consider the following indicators:

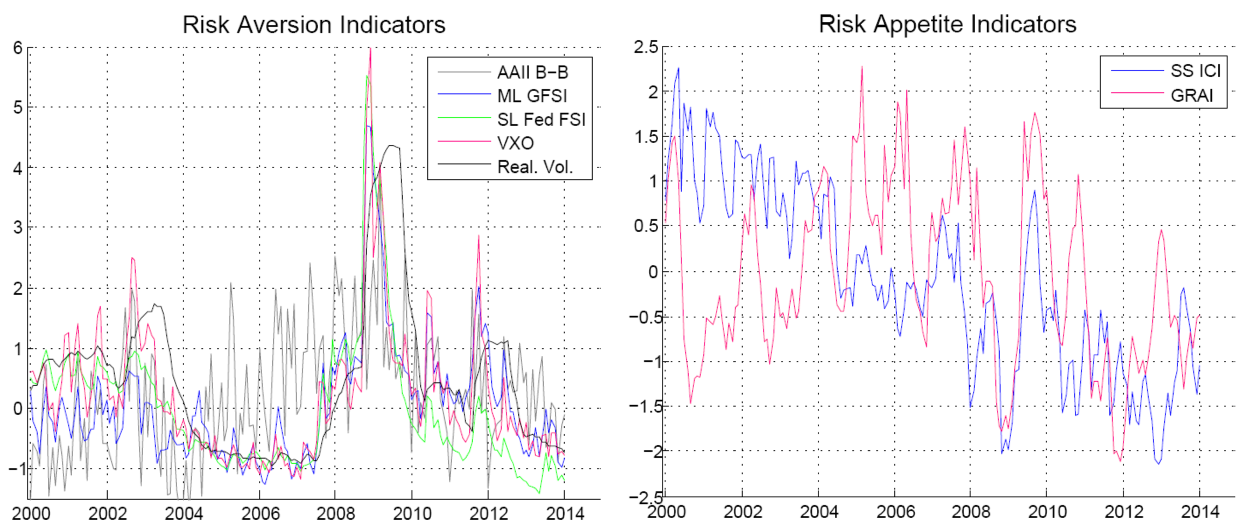
- The State Street Investor Confidence Index (SS ICI). It measures investor confidence or risk appetite quantitatively by analyzing the actual buying and selling patterns of institutional investors. The index assigns a precise meaning to changes in investor risk appetite: the greater the percentage allocation to equities, the higher risk appetite or confidence. It is available on a monthly basis from July 1998 onwards.
- The American Association of Individual Investors Bear Index minus Bull Index (AAII B-B). The indices reflect the sentiment of individual investors towards the stock market over the next 6 months. The question asked is "I feel that the direction of the stock market over the next 6 months will be…" (Bull, Bear and Neutral are the only possible answers). The American Association of individual Investors (AAII) Polls indicate the bullishness and bearishness of the stock market. It is available on a weekly basis from July 1987 onwards.
- The Bank of America - Merrill Lynch Global Financial Stress Index (ML GFSI). It is a cross-market gauge of risk, hedging demand and investment flows. The index is designed to help investors identify market risks. The GFSI composite index aggregates over twenty measures of stress across five asset classes and various geographies, measuring three separate kinds of financial market stress: risk, as indicated by cross-asset measures of volatility, solvency and liquidity; hedging demand, implied by the skew of equity and currency options; and investor appetite for risk, as measured by trading volumes as well as flows in and out of equities, high-yield bonds and money markets. It is available on a daily basis from January 2000 onwards.
- The Saint-Louis Federal Reserve Bank Financial Stress Index (SL Fed FSI). It uses 18 weekly data series to measure financial stress in the market: seven interest rate series, six yield spreads and five other indicators, including the VIX index. Each of these variables captures some aspect of financial stress. Accordingly, as the level of financial stress in the economy changes, the data series are likely to move together. It is available on a weekly basis from January 1994 onwards.
- The VIX. It is equal to the square root of a weighted average of two short maturity variance swap rates on the S&P 500 index, computed using a panel of option prices. It measures the annualized

expected volatility in the S&P 500 index over the upcoming 30-day period. It is available on a daily basis from January 1990 onwards.

- The One year realized volatility of the S&P 500 (Real. Vol.). It is computed on daily returns and hence available at a daily frequency from 1972 onwards.
- The Credit Suisse Global Risk Appetite Index (GRAI). The index compares risk (past price volatility) and excess returns across assets. The value of the CSFB on a given day is the slope coefficient obtained from the cross-sectional linear regression of risk and excess returns. The more positive the slope, the greater the risk appetite. It is available at a daily frequency from 1981 onwards.

All indices are normalized so that they have zero mean and unit standard deviation. We use the first dataset to filter the data because it is the one with the lowest frequency. We then figure out the corresponding values of the other datasets (we retain the first value available at the same time or after the release of the SS ICI). The monthly time-series of the indicators are plotted below for risk aversion indicators (left graph) and risk appetite indicators (right graph).

Figure 1 : Plot of risk aversion and risk appetite indicators



These graphs show the time-series of the risk aversion and risk appetite indicators. The series are normalized so to have zero mean and unit variance.

We observe that the risk aversion indicators strongly increase during the 2008 turmoil, while it is the opposite for the risk appetite indicators. The AAII Bear index minus Bull index fluctuates significantly and strongly lacks persistence. To illustrate this fact, we have computed the one-month and two-month autocorrelations of the series in Table 4 (Panel A).

Table 4: Correlation statistics for the investor sentiment variables

Index	SS ICI	AAII B-B	ML GFSI	SL Fed FSI	VIX	Real. Vol.	GRAI
PANEL A: Autocorrelation							
One-month autocorrelation	0.925	0.413	0.9	0.951	0.846	0.983	0.876
Two-month autocorrelation	0.853	0.385	0.778	0.877	0.68	0.943	0.736
PANEL B: Cross-correlation							
AAII B-B	-0.332						
ML GFSI	-0.334	0.423					
SL Fed FSI	0.075	0.298	0.816				
VIX	-0.064	0.353	0.864	0.869			
Real. Vol.	0.051	0.184	0.683	0.734	0.73		
GRAI	0.244	-0.066	-0.412	-0.247	-0.36	-0.186	

The statistics are computed on the period for which all variables are available (2000-2014). Auto-correlation is computed as the correlation between the variable and its one or two month lagged values while cross-correlation is simply the sample correlation between the variables.

The realized volatility is the indicator with the highest persistence because it relies on one year of past data which implies a strong memory effect. In contrast, the AAII B-B displays an autocorrelation which is much less than twice smaller. The VIX shows intermediate persistence between these two extremes. A high degree of persistence is obviously desirable in a context where we wish to use the variable to identify the prevalence of a given regime: indeed if the sentiment indicator signals a regime change and if a regime change lead to a change in portfolio composition, then the regime change ought to be persistent for the change in portfolio to yield positive returns in the future.

We provide in Panel B of Table 4 the correlation between the indicators. The State Street Investor Confidence Indicator is positively correlated to the GRAI, negatively correlated with two risk aversion indicators and almost uncorrelated to the remaining three. Four indices (ML GFSI, SL Fed, VIX and realized volatility) seem to carry redundant information because their correlations lie between 0.68 and 0.87.

In addition to realized volatility which will often be used as a state variable, we will report results based on the GRAI index because it carries information not contained in volatility and because the available sample is sufficiently large (1981 onwards) to capture long-term conditional behaviors.

3.2. Conditional performance of Smart Factor Indices

As we have discussed, market conditions such as bullish or bearish markets, as well as periods of high or low stock market volatility, happen to have a considerable impact on how different equity strategies perform. Amenc et al. (2012) find considerable variation in the performance of some popular smart beta strategies in different sub-periods, and show that certain market conditions favor some smart beta strategies while they prove detrimental to others.

In this section, we aim to characterize the conditional performance of factor indices based on the variables mentioned in Section 3.1.

3.2.1. Data

The factor indices originate from the Scientific Beta database (www.scientificbeta.com). Each factor consists of a long-short portfolio of the largest 500 US companies (S&P 500 universe). The portfolios are rebalanced quarterly to adjust for changes in the composition of the universe and the composition of the factors is set once every year. The size factor is long in the 250 stocks with smallest capitalizations (which are mid-cap stocks since no small cap stocks are to be found in the S&P500 universe) and short in the 250 stock with largest capitalizations. The value factor is long in the 250 stocks with the highest book-to-market ratios and short in the 250 stocks with the lowest book-to-market ratios. The momentum factor is long in the 250 stocks with the highest past performance over the past year, excluding the most recent month and short in the 250 stocks with the lowest performance over the same period of time. Lastly, the volatility factor is long in the 250 stocks with the lowest volatility over the past 2 years and short in the 250 stocks with the highest 2 year volatility. The portfolios are equally-weighted, subject to minor liquidity adjustments and returns include dividend payments (we use series of total returns). The sample starts in January 1972 and ends in December 2013.

The variables which are likely to impact the performance of these indices are listed in Section 3.1. The proxies we will use for these variables are listed in Table 5. All variables are available at the monthly frequency from 1972 to 2013, except liquidity (1985-2013) and GRAI (1981-2013).

Table 5 : Conditioning variables

Variable	Definition	Source
Credit spread	Moodys Baa minus Aaa corporate bond rates	Bloomberg
Term spread	10Y minus 3M US Treasury constant maturity rate	Bloomberg
Interest rates	1Y US Treasury constant maturity rate	Bloomberg
Liquidity	Minus the TED spread: 3M T-Bill minus 3M LIBOR rates	Bloomberg
Eco. Growth	US Chicago FED National Activity Indicator	Datastream
Inflation	Year-on-year variation of CPI	R. Shiller website
NBER cycles	NBER expansions and recession points	nber.org/cycles.html
GRAI	Global Risk Appetite Indicator	Crédit Suisse

The additional variables are computed using the factor returns. Volatilities are calculated as standard deviations of daily returns and performance is defined as the return over the specified period. The market factor corresponds to the returns of the S&P 500 (i.e., the portfolio of all stocks, weighted according to their capitalization).

3.2.2. Protocol and Results

For all of the variables discussed in section 3.1, we split the sample into the months for which the variable is above its median value and those for which it is below its median value (see Section 3.2.3. for an analysis of the robustness of the results with respect to a split in 3 states, as opposed to 2 states) and we compute the corresponding conditional average (monthly) return of the factor over these two subsamples. The variable is thus relevant if the two values are significantly different. In order to assess this, we perform the following *t*-test. Let $r_t^{\pm}$ denote the series of returns corresponding to months for which the state variable is above (+) its median or below (-) its median. The sample size on which the averages are computed is denoted *N*. We use the following *t*-statistic:

$$t = \frac{\mu^{+} - \mu^{-}}{\sigma^{+}/\sqrt{N}},$$

where the $\mu^{\pm}$ and $\sigma^{\pm}$ are straightforwardly the mean and standard deviation of the series of returns $r_t^{\pm}$ in the corresponding regimes. The statistic *t* is assumed to follow a law of Student with *N-1* degrees of freedom. We will use the notation stating that a *p*-value below 1% is associated with (***), a *p*-value between 1% and 5% with (**) and a *p*-value between 5% and 10% with (*). Note that the alternative (symmetric) test based on $t = \frac{\mu^{-} - \mu^{+}}{\sigma^{-}/\sqrt{N}}$ gives very similar results because the two standard deviations often differ at most of a factor 1.5.

3.2.2.1. *Size*

For the sake of brevity, we only report in Table 6 the variables that led to significant impacts on the performance of factors, but we comment on all variables below.

As expected, the size premium is strongly related to market movements and performs better when aggregate volatility is low. Moreover, our results are coherent with the lead-lag effect of Lo and MacKinlay (1990): the size premium is larger after the market has risen. When volatility is high, investors' risk appetite decreases, and the flight to quality effect implies that they favor large (safer) stocks.

In unreported results, we find no connection between conditional performance of the size factor and the NBER cycles. However, in our sample, small firms are outperformed by large firms during recoveries (high economic growth, low inflation), which is consistent with the results of Ilmanen (2011) who also underlines equities' forward-looking nature and countercyclical risk premium.

High term spreads are associated with significantly higher performance of the size factor. This is consistent with the fact that a positive slope for the yield curve is associated with upcoming favorable economic conditions, which positively affect small firms. We underline that in unreported results, neither interest rates, nor variations in interest rates have been found to impact the size premium in significant proportions. Nevertheless, we do find that favorable liquidity conditions are linked to above average outperformance of small firms. We also observe a positive link between the credit spread and the returns of the size factor. This is somewhat in line with the higher positive exposure of small firms (compared to large firms) that Fama and French (1993) report. Hahn and Lee (2006) show that size returns are negatively impacted by variations of the credit spread: small firms are more negatively impacted by contemporary *variations* of the credit spread than large firms. However, the relationship reverses when using the *level* of the credit spread as a conditioning variable.

Table 6 : Conditional performance of the size premium (variables stemming from the literature)

	N	Monthly return	Significance	Intuition
Low current market perf	252	-0.115	(***)	Beta >1
High current market perf	252	0.572		
Low current market vol	252	0.336	(**)	Flight to quality
High current market vol	252	0.121		
Low past 1M market perf	251	-0.115	(***)	Lead-lag effect
High past 1M market perf	251	0.572		
Economic recovery	169	0.027	(**)	Countercyclicality
Economic boom, slump or stagflation	335	0.330		
Low term spread	253	0.082	(***)	Positive economic outlook
High term spread	251	0.375		
Low credit spread	252	0.047	(***)	Size as proxy for distressed factor
High credit spread	252	0.410		
Low liquidity	175	-0.133	(***)	Favorable credit conditions
High liquidity	174	0.420		
Low past 1Y factor perf	246	0.081	(***)	Style momentum effect
High past 1Y factor perf	246	0.420		
Low current factor vol	252	0.126	(**)	Risk/return relationship
High current factor vol	252	0.331		

The sample period is split in two: months for which the variable is above its full-sample median and months for which it is below. Economic recovery is defined as a month for which US economic growth is above its median value and inflation is below its median value. The reported figures are the average monthly returns. Statistical significance is based upon the aforementioned *t*-statistic and the corresponding *p*-value: (***) when the *p*-value is smaller than 1%, (**) when it lies between 1% and 5% and (*) when it is between 5% and 10%.

The style momentum effect holds empirically for the size premium: past positive performance signals on average future positive performance. Moreover, similarly as in Li et al. (2009) for the value factor, the returns of size factor are positively related with the current level of their volatility.

Some indicators mentioned in the above table cannot be used for factor rotation strategies because there are synchronous with the realized returns, and are not predictable. A simple solution which overcomes this obstacle is to resort to lagged (past) data. Accordingly, we report additional results based on lagged variables and indicators computed on one year rolling windows in Table 7. Instead of looking at contemporary market returns and market and factor volatility, we condition the returns based on past one year or one month values of these indicators. We also include investors' risk appetite as an additional variable.

Table 7: Conditional performance of the size premium (lagged variables)

	N	Monthly return	Significance	Intuition
Low past 1Y market perf	246	0.477	(***)	Recovery
High past 1Y market perf	246	0.023		
Low past 1M market vol	251	0.063	(***)	Recovery
High past 1M market vol	252	0.380		
Low past 1Y market vol	246	0.007	(***)	Recovery
High past 1Y market vol	246	0.493		
Low past 1Y factor vol	246	-0.092	(***)	Risk/return relationship
High past 1Y factor vol	246	0.593		
Low past 1M factor vol	251	-0.041	(***)	Risk/return relationship
High past 1M factor vol	252	0.483		

The sample period is split in two: months for which the variable is above its full-sample median and months for which it is below. The reported figures are the average monthly returns. Statistical significance is based upon the aforementioned *t*-statistic and the corresponding *p*-value: (***) when the *p*-value is smaller than 1%, (**) when it lies between 1% and 5% and (*) when it is between 5% and 10%.

Past market conditions impact the performance of the portfolios based on firm size but the effect is the opposite of synchronous market performance: the highest returns follow low market returns (both over the past month and over the past year) and volatile periods. Past volatility of the size factor is also related to significant differences in conditional performance, with an effect that is the opposite of the effect reported when the analysis is based on synchronous volatility. The window over which the volatility estimate is computed does not seem to matter much because results are similar for a one month or a one year period. We also observe that market volatility and factor volatility give similar results, but they are more pronounced for factor volatility. Lastly, in unreported results, we did not find any meaningful link between investors' risk appetite and the performance of the size premium.

In sum, we identify the following variables as relevant for the characterization of the time-varying nature of the size factor:

- Past one month market performance,
- Credit spread,
- Term spread,
- Liquidity,
- Past one year factor performance,
- Past one year market performance,
- Past one month factor volatility.

Even though the 4 versions of past volatility yielded promising results, the overlap between these variables is substantial and we therefore retain only one out of the four so as to avoid redundancies.

3.2.2.2. *Value*

We start by gathering in Table 8 the variables mentioned in the literature that yielded significant differences in performance for the value factor.

Table 8 : Conditional performance of the value premium (variables stemming from the literature)

	N	Monthly return	Significance	Intuition
Low current market perf	252	0.564	(***)	Countercyclical factor
High current market perf	252	0.025		
Low current market vol	252	0.407	(**)	Flight to quality
High current market vol	252	0.181		
Past 1M decrease in term spread	256	0.388	(*)	
Past 1M increase in term spread	248	0.197		
Low liquidity	175	0.070	(**)	Flight to quality
High liquidity	174	0.376		
Low past 1Y factor perf	246	0.096	(***)	Style momentum effect
High past 1Y factor perf	246	0.535		
Low current factor vol	252	0.166	(***)	Risk/return relationship
High current factor vol	252	0.422		

The sample period is split in two: months for which the variable is above its full-sample median and months for which it is below. The reported figures are the average monthly returns. Statistical significance is based upon the aforementioned *t*-statistic and the corresponding *p*-value: (***) when the *p*-value is smaller than 1%, (**) when it lies between 1% and 5% and (*) when it is between 5% and 10%.

Value returns are negatively related to market returns and to contemporary market volatility (this last relationship is similar to that observed for the size premium). However, the impact of past market performance and market volatility is not significant. In unreported results, we also find that NBER cycles, credit spreads and interest rates (level and monthly variation) have only a marginal influence on the performance of the value premium. We also noticed that cross-sectional variance was negatively related to value payoffs, but the magnitude of this relationship is too limited and is not reported.

The term spread is only meaningful when conditioning with respect to its past increments, but the relationship is not very significant and contradicts the economic intuition. Furthermore, we find that positive variations in liquidity lead to lower performance of the value premium (not reported because statistically insignificant), which is in line with the findings of Asness et al. (2013). However, when taking the level of liquidity into account, the impact reverses: the returns of the value strategy are higher on average when liquidity is above its median value.

Similarly to the size factor, the value factor is positively related to its past performance over the previous year (style momentum effect) and also positively impacted by its synchronous volatility (which seems consistent with Li et al. (2009), even though the empirical protocols share little in common).

In continuation to our analysis, we report additional results based on lagged variables and indicators computed on one year rolling windows in Table 9. We also include investors' risk appetite in the set of variables.

Table 9: Conditional performance of the value premium (lagged variables)

	N	Monthly return	Significance	Intuition
Low past 1M market vol	251	0.378	(*)	Recovery
High past 1M market vol	252	0.206		
Low past 1Y factor vol	246	0.233	(*)	Risk/return relationship
High past 1Y factor vol	246	0.398		
Low past 1M factor vol	251	0.150	(***)	Risk/return relationship
High past 1M factor vol	252	0.433		
Low GRAI	198	0.166	(*)	Flight to quality
High GRAI	198	0.391		Appetite for risk

The sample period is split in two: months for which the variable is above its full-sample median and months for which it is below. The reported figures are the average monthly returns. Statistical significance is based upon the aforementioned *t*-statistic and the corresponding *p*-value: (***) when the *p*-value is smaller than 1%, (**) when it lies between 1% and 5% and (*) when it is between 5% and 10%.

The higher returns of the value strategy conditioned on synchronous volatility of the factor also hold when conditioning on past volatility of the factor. Given that volatility is a persistent , then this is consistent with the conclusions of Li et al. (2009). The results are more pronounced when looking only one month backward instead of one whole year. Past market volatility and performance yielded no meaningful impact when computed over 12 months (unreported), and are only mildly significant one when volatility is evaluated over a one-month rolling window.

Lastly, value payoffs are higher when investor risk appetite is high, even though this effect is only mildly significant. While the theoretical groundings of this relationship remain unclear, we can identify a flight to quality phenomenon: when risk appetite is low, investors favor the safer, glamorous (i.e. growth) stocks.

Overall, we notice that no conditioning variable leads to negative average returns for the value strategy. We propose the following variables for the conditioning of the value factor:

- Past one year performance of the value factor,
- Past one month volatility of the value factor,
- Liquidity,
- GRAI.

3.2.2.3. *Momentum*

The variables mentioned in Section 3.1.2.3 were tested and those that appeared significant are listed in the table below (Panel A).

Table 10 : Conditional performance of the momentum factor

	N	Monthly return	Significance	Intuition
PANEL A: Variables stemming from the literature				
Low past 1Y market perf	246	0.001	(*)	Recovery
High past 1Y market perf	246	0.288		
Low past 1Y market vol	246	0.427	(***)	Recovery
High past 1Y market vol	246	-0.138		
Momentum crash	127	-0.405	(**)	Momentum crash
Non momentum crash	365	0.336		
Low current factor vol	246	0.438	(***)	Reverse risk/return relationship
High current factor vol	246	-0.150		
PANEL B: Lagged variables				
Low past 1M factor vol	251	0.460	(***)	Reverse risk/return relationship
High past 1M factor vol	252	-0.158		
Low past 1Y factor vol	246	0.439	(***)	Reverse risk/return relationship
High past 1Y factor vol	246	-0.150		
Low GRAI	198	-0.145	(**)	
High GRAI	198	0.322		

The sample period is split in two: months for which the variable is above its full-sample median and months for which it is below. Momentum crash is defined as a month for which past 1Y market performance is below its median value and past 1Y market volatility is above its median value. The reported figures are the average monthly returns. Statistical significance is based upon the aforementioned *t*-statistic and the corresponding *p*-value: (***) when the *p*-value is smaller than 1%, (**) when it lies between 1% and 5% and (*) when it is between 5% and 10%.

Momentum profits are positively related to past market returns, but negatively related to past aggregate volatility. When combining both effects, we recover the momentum crashes, which are subsequent to strong market downturns. The seemingly lower significance is an artifact of the smaller sample size of the momentum crashes. In addition, even though cross-sectional variance displayed a negative relation with momentum payoffs (consistently with the results of Sun and Stivers (2010)), this relation was not outstanding and is not mentioned in the table above.

In unreported results, we also found that momentum returns are slightly higher when liquidity increases (as in Asness et al. (2013)), but the effect is not statistically significant (this is also the case for the credit spread, which we do not report either).

Furthermore, unlike for the size and value factors, the phenomenon of style momentum does not hold empirically for the momentum factor. A possible explanation for this is that a momentum effect for a momentum factor would imply that some kind of a stronger and longer momentum than what is originally present in the underlying time-series.

In contrast to the size and value factors, the momentum exhibits a positive relationship between its returns and its synchronous volatility. As such, the momentum factor cannot be considered as a risk factor (Charoenrook and Conrad (2008)). In fact, most theories put forward to explain the momentum premium are behavioral rather than risk-based.

In Panel B, we report additional results based on lagged variables and indicators computed on one year or one month rolling windows. Factor-based volatilities lead to similar, although more pronounced, discrepancies, compared to market volatility. The risk appetite index yields significant spreads in performance, and this effect could be linked to the momentum crash phenomenon since investor risk appetite is usually low after market crashes.

Overall, we retain the following variables for the conditioning of the momentum factor:

- Past 1Y market performance,
- Past 1Y market volatility,
- Past 1M factor volatility,
- GRAI.

3.2.2.4. *Volatility*

We proceed similarly with the volatility-based factor and report the conditional average returns in Table 11. In panel A, we focus on the variable which originate from our literature review.

Straightforwardly, low volatility (hence low beta) stocks underperform their high volatility counterparts in bullish markets. The impact is strongly significant because the long-short factor has a beta well below zero. The factor is shown to perform much better when volatility is high (and when investors seek safer assets) and when interest rates are decreasing: Coqueret et al. (2014) show that a low volatility minus high volatility portfolio is positively correlated to bond returns which are in turn negatively exposed to changes in interest rates.

In Panel B, we deal with lagged variables which are those that can most easily be used in practice in factor rotation strategies in the absence of predictability of the conditioning variable. While the results continue to hold for the lagged variation in interest rates (though the pattern is less marked), the performance reverses for the lagged volatilities. Compared to synchronous volatility, a reversal occurs and past high volatility signals lower returns than past low volatility. The effect remains nonetheless significant and is valid across sample size (1Y and 1M) and across underlying (market or factor).

Table 11 : Conditional performance of the volatility factor

	N	Monthly return	Significance	Intuition
PANEL A: Variables stemming from the literature				
Low current market perf	252	1.400	(***)	Beta <1
High current market perf	252	-1.436		
Low current market vol	252	-0.345	(**)	Flight to quality
High current market vol	252	0.309		
Decreasing interest rates	252	0.545	(***)	Bond Similitude
Increasing interest rates	252	-0.570		
PANEL B: Lagged variables				
Low past 1M market vol	251	0.106	(*)	Negative exposure to volatility
High past 1M market vol	252	-0.121		
Low past 1Y market vol	246	0.153	(***)	Negative exposure to volatility
High past 1Y market vol	246	-0.216		
Low past 1M factor vol	251	0.180	(***)	Negative exposure to volatility
High past 1M factor vol	252	-0.194		
Low past 1Y factor vol	246	0.134	(**)	Negative exposure to volatility
High past 1Y factor vol	246	-0.196		
Past fall in interest rates	251	0.288	(***)	Bond Similitude
Past rise in interest rates	252	-0.330		

The sample period is split in two: months for which the variable is above its full-sample median and months for which it is below. The reported figures are the average monthly returns. Statistical significance is based upon the aforementioned *t*-statistic and the corresponding *p*-value: (***) when the *p*-value is smaller than 1%, (**) when it lies between 1% and 5% and (*) when it is between 5% and 10%.

In conclusion, the following variables are good drivers of the conditional performance of the volatility-based factor:

- Past 1Y market volatility,
- Past 1M factor volatility,
- Past 1M variation in interest rates.

3.2.3. Robustness Checks

As a robustness check, we split the sample into three instead of two and observe if the average returns are monotonous across the terciles of the variables. Accordingly, we translate the previous protocol to a tercile analysis and display the results for all four factors. For the size factor, the results are detailed in Table 12.

Table 12 : Conditional performance of the size factor (tercile split)

	N	Monthly return	Significance	Intuition
Low past 1M market perf	168	-0.040		
Medium past 1M market perf	167	0.110	(***)	Lead-lag effect
High past 1M market perf	168	0.600		
Low past 1Y market perf	164	0.677		
Medium past 1Y market perf	164	0.100	(***)	Recovery
High past 1Y market perf	164	-0.026		
Low term spread	168	0.047		
Medium term spread	168	0.159	(***)	Positive economic outlook
High term spread	168	0.479		
Low credit spread	162	0.015		
Medium credit spread	164	0.107	(***)	Size as a proxy for distressed factor
High credit spread	178	0.534		
Low liquidity	116	-0.186		
Medium liquidity	117	0.023	(***)	Favorable credit conditions
High liquidity	116	0.592		
Low past 1Y factor perf	164	-0.005		
Medium past 1Y factor perf	164	0.102	(***)	Style momentum effect
High past 1Y factor perf	164	0.654		
Low past 1M factor vol	168	-0.105		
Medium past 1M factor vol	167	0.150	(***)	Risk/return relationship
High past 1M factor vol	168	0.620		

The sample period is split in three: months for which the variable is above its full-sample high tercile, months for which it is below the low tercile and months for which it is in between. The reported figures are the average monthly returns. Statistical significance is assessed between high and low values of the variable and is based upon the aforementioned *t*-statistic and the corresponding *p*-value: (***) when the *p*-value is smaller than 1%, (**) when it lies between 1% and 5% and (*) when it is between 5% and 10%.

We observe a monotonous pattern for all seven variables: returns are a strictly increasing function of the variables, except for low past 1Y market performance, which is associated to strictly decreasing returns for the size factor. Moreover, the spreads in returns between months related to high values of the variables and those related to low values are all strongly significant. As such, these results confirm the relevance of the selected variables for the factor based on market capitalization. The corresponding results for the value factor are gathered in Table 13.

Table 13 : Conditional performance of the value factor (tercile split)

	N	Monthly return	Significance	Intuition
Low GRAI	130	0.081		Flight to quality
Medium GRAI	134	0.316	(**)	
High GRAI	132	0.436		Appetite for risk
Low liquidity	116	-0.078		
Medium liquidity	117	0.355	(***)	Favorable credit conditions
High liquidity	116	0.389		
Low past 1Y factor perf	164	0.066		
Medium past 1Y factor perf	164	0.307	(***)	Style momentum effect
High past 1Y factor perf	164	0.574		
Low past 1M factor vol	168	0.059		
Medium past 1M factor perf	167	0.414	(*)	Risk/return relationship
High past 1M factor vol	168	0.404		

The sample period is split in three: months for which the variable is above its full-sample high tercile, months for which it is below the low tercile and months for which it is in between. The reported figures are the average monthly returns. Statistical significance is assessed between high and low values of the variable and is based upon the aforementioned *t*-statistic and the corresponding *p*-value: (***) when the *p*-value is smaller than 1%, (**) when it lies between 1% and 5% and (*) when it is between 5% and 10%.

The figures show that three out of four variables lead to a monotonous pattern in average returns of the value factor. The past one month factor volatility fails to deliver monotonicity, but is not far from it either. Furthermore, the discrepancy between factor return during low factor volatility periods and high factor volatility periods is significant at the 10% level. Consequently, these results confirm our initial choice of variables for the conditioning of the returns of the value factor. We perform the same analysis for the momentum factor in Table 14.

Similarly to the value factor, one variable (the Global Risk Aversion Index) out of four does not imply monotonous average returns. Nevertheless, the spread between returns related to high GRAI and those related to low GRAI is significant at the 1% level. The first two variables, related to the momentum crash phenomenon, yield monotonous returns and exhibit a strong contrast between the two extreme terciles. Overall, these results are consistent with the findings of Section 3.2. Lastly, we turn to the conditional performance of the volatility-based factor in Table 15.

Table 14 : Conditional performance of the momentum factor (tercile split)

	N	Monthly return	Significance	Intuition
Low past 1Y market perf	164	-0.238		
Medium past 1Y market perf	164	0.251	(***)	Beta > 0
High past 1Y market perf	164	0.420		
Low past 1Y market vol	164	0.442		
Medium past 1Y market vol	164	0.243	(**)	Beta > 0
High past 1Y market vol	164	-0.251		
Low past 1M factor vol	168	0.567		
Medium past 1M factor vol	167	-0.010	(**)	Reverse risk/return relationship
High past 1M factor vol	168	-0.107		
Low GRAI	130	-0.361		Flight to quality
Medium GRAI	134	0.345	(***)	
High GRAI	132	0.270		Appetite for risk

The sample period is split in three: months for which the variable is above its full-sample high tercile, months for which it is below the low tercile and months for which it is in between. The reported figures are the average monthly returns. Statistical significance is assessed between high and low values of the variable and is based upon the aforementioned *t*-statistic and the corresponding *p*-value: (***) when the *p*-value is smaller than 1%, (**) when it lies between 1% and 5% and (*) when it is between 5% and 10%.

We acknowledge that only one variable generates monotonicity among average returns and a strong significance between extreme terciles. For the other two variables, not only is the pattern far from monotonic, but the upper and lower terciles are related to average returns that are not significantly distinguishable. Accordingly, we will perform robustness checks in Section 3.3.3.5. where these variables will be omitted.

Table 15 : Conditional performance of the volatility-based factor (tercile split)

	N	Monthly return	Significance	Intuition
Low past 1Y market vol	164	0.017		Negative exposure to volatility
Medium past 1Y market vol	164	0.106		
High past 1Y market vol	164	-0.217		
Low past 1M factor vol	168	0.196		
Medium past 1M factor vol	167	-0.073	(**)	Negative exposure to volatility
High past 1M factor vol	168	-0.146		
Past 1M decrease in interest rates	166	0.240		
Past 1M stability in interest rates	170	-0.321		Bond-like behavior
Past 1M increase in interest rates	168	0.034		

The sample period is split in three: months for which the variable is above its full-sample high tercile, months for which it is below the low tercile and months for which it is in between. The reported figures are the average monthly returns. Statistical significance is assessed between high and low values of the variable and is based upon the aforementioned *t*-statistic and the corresponding *p*-value: (***) when the *p*-value is smaller than 1%, (**) when it lies between 1% and 5% and (*) when it is between 5% and 10%.

3.3. Implications for Tactical Allocation Decisions to Smart Factor Indices

In this Section, we investigate the implications of the presence of predictability in smart factor index performance as a function of the market environment for active allocation decisions to smart factor indices. In a nutshell, the goal is to out-perform the strategic benchmark (equally-weighted benchmark or relative equal risk contribution benchmark) from Section 2 by a systematic over/underweighting of strategies based on their expected out/under-performance given different market conditions. It is important to emphasize again at this stage that the list of factors used in the analysis will be limited to the factors composing the strategic benchmark. In particular, only rewarded tilts will be included. As a result, if say the value index is expected to under-perform in a given market environment, we shall decrease its allocation with respect to the strategic benchmark allocation, and potentially set it to zero, as opposed to allocating to the growth index, or shorting the value index, which would not be practically feasible.

We carry out an allocation exercise over smart factor indices based on the aforementioned database (US data on the sample period 1972-2013). The portfolios are rebalanced on a monthly basis and weights are computed as a pre-specified function of the regimes. A first heuristic approach is to determine particular allocations depending on regimes.

A typical such example would be to underweight the low volatility tilts in periods of market tranquility and overweight them in phases of turbulence. This simple technique is implemented by Kritzman et al. (2012) in the case of multi-asset strategies and by Ammann and Verhofen (2006) and Bulla et al. (2011), who propose a brutal switch from equity indices to risk-free assets in volatile regimes. The specifications of the allocation step depend critically on the results of the previous subsection, but they can be formally be detailed as follows.

3.3.1. Translating Conditional Signals into Portfolio Weights

In accordance with the notations of the previous section, we denote by $w(i,t)$ the weight of index *i* at time *t*. It is obvious that if *t* falls within regime *j*, and if it has been shown that the average returns satisfy $AR(i_1,j) \geq AR(i_2,j)$, then this should imply $w(i_1,t) \geq w(i_2,t)$. This simply means that we allocate more to the indices that are expected to perform well in regime *j*.

Our first (heuristic) approach proceeds as follows. From the results of the previous section, we infer that for any given relevant variable, we can determine whether a month will be favorable or unfavorable to a particular factor index. Given the results of Table 6 to Table 11, we build signals $s_{v,t}^F$ which are equal to one if the variable *v* is associated to expected outperformance of factor *F* at time *t* and equal to zero otherwise. We then aggregate the signals at the factor level by combining all of the relevant variables for this factor:

$$I(F,t) = \frac{1}{n_F}\sum_{i=1}^{n_F} s_{i,t}^F,$$

where n_F is the number of relevant variables for factor *F*. By construction, the intensity *I(F,t)* lies between 0 and 1: a zero intensity is associated to an expected strong underperformance while a value of one signals very favorable conditions for the factor. Based on this intensity, we derive a binary "Go" or "No Go" signal:

$$G(F,t) = \begin{cases} 1 \text{ if } G(F,t) \geq 0.5 \\ k \text{ if } G(F,t) < 0.5 \end{cases}$$

and the multiplicative factor $k \in (0,1)$ modulates the discrepancy between the indices which are expected to outperforms and those with below-average conditional performance. This factor can either been seen as a parameter reflecting aggressiveness or confidence in active views regarding the chosen indices. Indeed, a *k* close to zero will imply that the outstanding indices will be strongly overweighed. Accordingly, several values of *k* should be tested.

In order to test the relevance of the intensity and go/no-go signals, we compute the in-sample returns related to each signal, for each factor in Table 16.

Table 16 : Conditional average return based in intensity and go/no-go signals

Long-Short	Size		Val/Growth		Momentum		Volatility	
	Panel A: Intensity							
Low intensity	0	-1.64%	0	-0.08%	0	-0.87%	0	-1.13%
	1	-0.60%	1	-0.02%	1	0.18%	1	0.24%
	2	-0.16%	2	0.30%	2	0.26%	2	0.11%
	3	0.24%	3	0.37%	3	0.28%	3	0.26%
	4	0.43%	4	1.30%	4	0.74%		
	5	0.61%						
	6	1.11%						
High intensity	7	0.65%						
	Panel B: Binary							
No go	0	-0.19%	0	-0.02%	0	-0.20%	0	-0.24%
Go	1	0.57%	1	0.47%	1	0.34%	1	0.20%
Long-Only	Mid Cap		Value		Winners		Low vol	
	Panel A: Intensity							
Low intensity	0	0.30%	0	0.31%	0	1.32%	0	1.43%
	1	0.87%	1	1.26%	1	0.82%	1	0.75%
	2	0.74%	2	1.79%	2	1.12%	2	1.05%
	3	1.05%	3	0.95%	3	1.62%	3	1.76%
	4	1.44%	4	1.34%	4	1.23%		
	5	1.58%						
	6	2.60%						
High intensity	7	1.68%						
	Panel B: Binary							
No go	0	0.83%	0	1.04%	0	0.96%	0	0.98%
Go	1	1.60%	1	1.42%	1	1.35%	1	1.28%

The sample period is split into periods according to the values of the intensity (*I(F,t)*) or the go/no-go (*G(F,t)*) signals. The number of periods is factor-dependent because it is determined by the number of relevant variables determined in Section 3.2. The first half of the table focuses on long-short portfolios while the second half deals with the (long-only) rewarded legs of the factors. The reported figures are the average monthly returns.

The average returns for the long-short factors are monotonous for the value versus growth tilts and winners minus loosers (momentum), and close to monotonous for the other two factors based on market capitalization and volatility. We also note a strong discrepancy between the average returns associated to "Go" and "No-Go" periods, confirming the relevance of the chosen variables for building active signals. The results for the long-only factors are less pronounced but broadly follow overall the same patterns.

In what follows, we use the signals *I(F,t)* and *G(F,t)* to build portfolio weights (for the factors) in the following fashion: the weight in each factor is proportional to its intensity or go/no-go signal:

$$w_I(F,t) = \frac{I(F,t)}{\sum_{F=1}^{4} I(F,t)}, \qquad \text{or} \qquad w_G(F,t) = \frac{G(F,t)}{\sum_{F=1}^{4} G(F,t)}.$$

While this heuristic approach is well-suited when the strategic benchmark is an equally-weighted portfolio of the chosen factors. A pragmatic extension to this heuristic approach in a situation when the strategic benchmark is an absolute or relative risk parity portfolio (see Section 2.2.) consists in using a formal risk budgeting portfolio construction methodology, where deviations from the equal risk contribution benchmarks are based on the perceived outperformance of selected factor tilts in various market conditions.

When the strategic benchmark is based on risk parity (in absolute or relative terms), it amounts to equate the risk contributions of each asset: $RC_i = 1/N$. It is possible to include active views to this framework by allowing a non-uniform distribution of risk contributions. We recall the risk decomposition associated to portfolio *w*:

$$\sigma(w) = \sum_{i=1}^{4} w_i \frac{[\Sigma w]_i}{\sigma(w)} = \sum_{i=1}^{4} RC_i \, .$$

Based on this representation, we want to allocate the risk budget according to the views in the following way:

$$\frac{RC_F}{\sigma(w)} = w_I(F,t) \qquad or \qquad \frac{RC_F}{\sigma(w)} = w_G(F,t),$$

where the risk contributions are normalized, so that, like the weights, they sum to one. Note that these representations must then be translated into portfolio weights via a numerical optimization. We further highlight that this risk budgeting approach can be performed on raw returns or on relative returns, i.e. returns net of those of a chosen benchmark.

3.3.2. Protocol and Results

The investment universe consists of four indices extracted from the Scientific Beta database: the midcap smart factor index, the value smart factor index, the momentum smart factor index and the low volatility smart factor index, all weighted according to the maximum deconcentration principle (which is very close to an equally-weighted scheme, up to liquidity constraints). As in the previous subsection, the dataset starts in January 1972 and ends in December 2013. Unless specified otherwise, the rebalancing occurs on a monthly basis. Numerous robustness checks will subsequently be performed to check for the impact of relaxing some of these assumptions.

The performance indicators which we will rely on are the following:

- *Annualized return*: $AR(P) = (P_T/P_1)^{1/y} - 1$, where the P_t are portfolio values, *T* is the sample size and *y* is the number of years in the sample (*y*=42).
- *Annualized volatility*: $Vol = std(r_t) \times \sqrt{12}$, where the r_t are the monthly returns of the portfolios.
- *Sharpe ratio*: $SR = (AR(P) - AR(rf))/Vol$, where *AR(rf)* is the annualized return of 3 months US treasury bills.
- *Maximum Drawdown*: maximum peak-to-trough decline of the value of the portfolio over the whole sample period.
- *Annualized racking error:* $TE(P,B) = std(r_t^P - r_t^B) \times \sqrt{12}$, where r_t^P are the monthly returns of the portfolio and r_t^B are the corresponding returns of a benchmark. Two benchmarks will be used: either the cap-weighted index of all stocks in the universe, or the equally-weighted portfolio of the 4 factor indices.
- *Information ratio* : $IR(P,B) = (AR(P) - AR(B))/TE(P,B)$.
- *Hit ratio* : the percentage of time that active views allow to outperform the heuristic benchmark (equally weighted portfolio of the 4 indices):

$$HR = \frac{1}{T}\sum_{t=1}^{T} 1_{r_t^P \geq r_t^B}.$$

We present the results of the simple allocation based on intensity and go/no-go (binary) signals in Table 17 below.

Table 17 : Performance of active allocation to factor indices

	Ann. Return	Vol	SR	MaxDD	TE (CW)	IR (CW)	TE (EW)	IR (EW)	HR(EW)
CW (stocks)	10.5%	15.7%	0.33	52.4%	-	-	5.81%	-0.60	44.6%
Intensity	15.4%	17.7%	0.57	56.0%	6.53%	0.76	1.90%	0.76	57.5%
Binary (k=0.00)	15.5%	18.2%	0.56	56.7%	7.04%	0.72	2.69%	0.58	65.1%
Binary (k=0.25)	14.9%	17.6%	0.54	55.8%	6.37%	0.69	1.46%	0.63	65.1%
Binary (k=0.50)	14.5%	17.3%	0.53	55.6%	6.09%	0.66	0.79%	0.66	65.1%
Binary (k=0.75)	14.2%	17.1%	0.52	55.7%	5.95%	0.62	0.34%	0.65	65.1%
EW (factors)	14.0%	17.0%	0.51	55.7%	5.87%	0.59	0.00%	-	-

The sample period is 1972-2013 and rebalancing is executed at a monthly frequency. The four tilted building blocks are all weighted according to the maximum deconcentration principle. The allocation process is driven by $w_I(F,t)$ for the intensity approach and by $w_G(F,t)$ for the binary method. The EW portfolio of all factors is equivalent to the binary method for k=1. The hit ratios related to the binary approach must be decomposed into the months with equal performance (19.3% of the sample) and month with strictly superior performance (45.8% of the sample).

The portfolios based on conditional information display higher returns and Sharpe ratios, not only when compared to the S&P500 CW portfolio but also when compared to the EW portfolio of the selected factors. This is at the cost of slightly higher volatilities and maximum drawdown. The tracking errors with respect to the CW benchmark are slightly superior for the actively managed factor portfolios compared to the strategic EW benchmark. Nevertheless, the higher levels of the corresponding information ratios indicate that this higher tracking error is more than compensated by the additional upside potential brought by the tactical views. The comparison to the EW benchmark confirms the added value of the tactical views because the information ratios are all positive, and the hit ratios are all well above 50%. Lastly, among the ways that the views are processed, the intensity approach appears to provide the best compromise between absolute *and* relative risk-adjusted performance.

The performance of the portfolios based on the absolute and relative (to the CW benchmark) risk budgeting technique are gathered in Table 18. Note that for the risk-budgeting approaches, the strategic benchmarks are the ERC portfolio in the case of absolute risk budgeting and the R-ERC portfolio for relative risk budgeting.

Table 18 : Performance of active allocation to factor indices – Risk budgeting approach

	Ann. Return	Vol	SR	MaxDD	TE (CW)	IR (CW)	TE (ERC)	IR (ERC)
CW (stocks)	10.5%	15.7%	0.33	52.4%	0.00%	-	5.81%	-0.60
Panel A: Absolute Risk Budgeting								
Intensity	15.4%	17.5%	0.57	55.7%	6.44%	0.76	2.01%	0.75
Binary (k=0.00)	15.5%	18.2%	0.56	56.7%	7.02%	0.71	2.85%	0.55
Binary (k=0.25)	14.8%	17.4%	0.54	55.5%	6.26%	0.69	1.48%	0.62
Binary (k=0.50)	14.4%	17.1%	0.53	55.3%	5.98%	0.65	0.79%	0.65
Binary (k=0.75)	14.1%	16.9%	0.52	55.3%	5.84%	0.62	0.33%	0.67
ERC	13.9%	16.8%	0.51	55.4%	5.77%	0.59	0.00%	-
Panel B: Relative Risk Budgeting							TE (R-ERC)	IR (R-ERC)
Intensity	15.2%	17.1%	0.57	55.4%	5.91%	0.79	2.12%	0.70
Binary (k=0.00)	15.2%	17.8%	0.55	56.8%	6.59%	0.71	2.99%	0.49
Binary (k=0.25)	14.5%	16.9%	0.54	54.5%	5.47%	0.72	1.23%	0.61
Binary (k=0.50)	14.1%	16.6%	0.53	54.5%	5.20%	0.69	0.63%	0.63
Binary (k=0.75)	13.9%	16.5%	0.51	54.6%	5.08%	0.67	0.26%	0.65
R-ERC	13.7%	16.5%	0.51	54.7%	5.02%	0.64	0.00%	-

The sample period is 1972-2013 and rebalancing is executed at a monthly frequency. The four tilted building blocks are all weighted according to the maximum deconcentration principle. The allocation process is driven by $w_I(F,t)$ for the intensity approach and by $w_G(F,t)$ for the binary method. The ERC and R-ERC portfolio of all factors are equivalent to the binary method for k=1 in Panel A and Panel B, respectively.

For the absolute risk budgeting approach, the figures are very close to those obtained with the heuristic weighting scheme. The maximum spread between the indicators is 10 basis points. The weights based on relative risk budgeting imply similar Sharpe ratios because the slightly lower returns are compensated by slightly lower volatilities. We nevertheless acknowledge that this latter method leads also to lower tracking errors, both with respect to the cap-weighted portfolio of all stocks and to the strategic benchmark of the four indices. While this reduction in tracking error is sufficient to ensure a higher information ratio with respect to the CW benchmark (compared to the heuristic allocation process), it is not the case when the relative performance is assessed with respect to the strategic benchmarks. Lastly, the extreme risk (maximum drawdown) is slightly increased compared to the CW benchmark (from 52% to 55%), but remains at the level of the strategic benchmark.

Overall, these first results show that, irrespectively of the chosen weighting scheme (heuristic or risk budgeting), the added value of tactical views can be substantial. When looking at the intensity-based procedure, the increase in annual return reaches 140 to 150 basis points while the volatility gains only half of this amount in absolute value. The Sharpe ratio increases from 0.51 to 0.57 and the improvement of the tracking errors is even more pronounced.

3.3.3. Robustness Checks

We now perform a number of robustness checks with respect to the choice of the weighting scheme used in the construction of the factor indices, the sample period under consideration, the rebalancing frequency, the use of a turnover reduction methodology or the use of terciles in the classification of the states of the market.

3.3.3.1. Alternative Index Weighting Schemes

Our base case study focuses on tilted indices weighted according to the maximum deconcentration principle because the primary goal was to identify favorable regimes for the factors and hence, non-uniform weights might have biased our results. Interestingly enough, though, we show that the superior performance displayed in the previous subsection translates to indices with alternative weighting schemes.

In order to do so, we duplicate our results to two additional weighting schemes. The first weighting scheme is based on market capitalization: once the stocks have been identified as belonging to a rewarded tilt, they are weighted proportionally to their capitalization (CW). The second approach (multi-strategy - MS) is based on a uniform blend of five optimized weighting schemes (efficient minimum volatility, efficient maximum Sharpe ratio, maximum decorrelation, maximum deconcentration and diversified risk-weighted).[2] The combination of multiple schemes allows the reduction the risk of model misspecification while providing enhanced diversification potential, compared to heuristic schemes.

We provide the usual indicators related to the allocation to the new sets of indices in Table 19.

[2] See www.scientificbeta.com for more details.

Table 19 : Alternative weighting schemes

	Ann. Return	Vol	SR	MaxDD	TE (CW)	IR (CW)	TE (EW)	IR (EW)	HR(EW)
CW (stocks)	10.5%	15.7%	0.33	52.4%	-	-	5.81%	-0.60	44.6%
Panel A: MS									
Intensity	15.5%	16.3%	0.62	52.0%	5.90%	0.84	1.60%	0.82	58.3%
Binary (k=0.00)	15.5%	16.8%	0.61	52.6%	6.27%	0.80	2.29%	0.60	64.9%
Binary (k=0.25)	15.0%	16.3%	0.59	51.9%	5.84%	0.77	1.23%	0.67	64.9%
Binary (k=0.50)	14.6%	16.1%	0.58	51.8%	5.68%	0.73	0.67%	0.69	64.9%
Binary (k=0.75)	14.4%	15.9%	0.56	51.9%	5.60%	0.69	0.28%	0.71	64.9%
EW (factors)	14.2%	15.9%	0.55	51.9%	5.56%	0.66	0.00%	-	-
Panel B: CW									
Intensity	13.5%	16.1%	0.51	53.1%	3.52%	0.86	1.93%	0.77	58.3%
Binary (k=0.00)	13.7%	16.7%	0.50	54.9%	4.49%	0.71	3.23%	0.50	67.7%
Binary (k=0.25)	13.0%	16.1%	0.48	53.2%	3.46%	0.73	1.68%	0.58	67.7%
Binary (k=0.50)	12.6%	15.8%	0.46	52.7%	3.11%	0.67	0.89%	0.61	67.7%
Binary (k=0.75)	12.3%	15.7%	0.44	52.5%	2.97%	0.60	0.38%	0.61	67.7%
EW (factors)	12.0%	15.6%	0.43	52.6%	2.92%	0.53	0.00%	-	-

The sample period is 1972-2013 and rebalancing is executed at a monthly frequency. The four tilted building blocks are all weighted according to either the CW or MS principle. The allocation process is driven by $w_I(F,t)$ for the intensity approach and by $w_G(F,t)$ for the binary method. The EW portfolio of all factors is equivalent to the binary method for k=1. The hit ratios related to the binary approach embed a 19.3% probability that the portfolio has the same allocation (and hence return) as the strategic benchmark.

A first observation is the spread in performance between the two EW strategies (220 basis points), which underlines the improved diversification benefits of the MS strategy over the CW scheme. When looking at the intensity-based method, we see that the improvements brought by the active views translate in a straightforward manner to the new sets of indices: the increase in performance reaches 130 basis points for the MS indices and 150 basis points for the CW indices. In the meantime, the volatility gains only 40 to 50 basis points. This implies superior Sharpe ratios for the portfolios built on the active views. Moreover, the hit ratios are, as in the base case study, well above 50%. All maximum drawdown figures lie in a very small window around between 51% and 54%.

The tracking errors with respect to the CW benchmark are naturally lower for the CW indices. Among these indices however, the lower tracking error of the EW benchmark does not lead to a higher information ratio compared to the intensity-based allocation. This is because the spread in performance is far from compensated by the gain in tracking error. Likewise, the value of the tactical views is highlighted when we examine the impact of *k*: the higher the *k*, the smaller the impact of the views. For both sets of indices, the Sharpe ratio decreases with *k*. Overall, similarly to the base case setup, the intensity-based technique leads to the best absolute and relative risk-adjusted performance.

Overall we find that the benefits of factor timing are maintained with CW factor indices, but the use of improved (also known as smart) factor indices lead to a much greater overall performance.

3.3.3.2. *Subperiod Analysis*

The results we have reported so far all cover our whole sample, which ranges from 1972 to 2013. It is a natural question to wonder what the performance indicators might be on smaller, specific, subperiods. This subsection is devoted to answering this question. To answer this question, we duplicate our results for the intensity methodology on heuristic subsamples (decades), on conditional samples (based on market performance) and on two particular macroeconomic cycles: the tech bubble and the subprime crisis. The results are reported in Table 20.

Table 20 : Performance during subperiods

	Ann. Ret.	Vol	SR	MaxDD	TE (CW)	IR (CW)	TE (EW)	IR (EW)	HR(EW)
ALL	15.4%	17.7%	0.57	56.0%	6.53%	0.76	1.90%	0.76	57.5%
197201-198206	11.9%	18.5%	0.20	35.2%	8.28%	0.79	2.39%	0.59	47.6%
198207-199212	22.6%	17.3%	0.89	31.1%	4.23%	0.98	0.93%	1.66	65.1%
199301-200306	14.6%	14.5%	0.72	24.5%	7.56%	0.59	1.94%	0.81	54.0%
200307-201312	13.0%	19.9%	0.57	56.0%	5.25%	0.86	2.06%	0.62	63.5%
Bull month	60.6%	12.2%	4.54	-	6.23%	1.28	1.95%	1.44	59.5%
Bear month	-31.6%	14.0%	-2.64	-	6.96%	0.31	1.80%	0.18	54.4%
Bull year	22.9%	16.5%	1.08	-	5.67%	0.69	1.72%	0.95	58.3%
Bear year	-8.2%	20.3%	-0.71	-	8.94%	0.83	2.46%	0.37	54.6%
Tech bubble (95-00)	20.7%	13.3%	1.18	16.3%	5.39%	-1.17	1.29%	1.25	55.7%
Subprimes (07-09)	-14.5%	34.9%	-0.46	55.0%	8.94%	0.14	3.84%	0.49	68.0%

The whole sample period is 1972-2013 and rebalancing is executed at a monthly frequency. The four tilted building blocks are all weighted according to the maximum deconcentration principle. The allocation process is driven by $w_I(F,t)$ (intensity approach). The indicators are computed as if all returns were consecutive. The 504 months of the full sample are subdivided into categories. The bull/bear year sets are those for which months belong to a bull or bear year, according to the CW benchmark performance over the calendar year. The tech bubble starts in January 1995 and ends in January 2000. The subprime crisis starts in July 2007 and ends in June 2009. The definition of all other subsets is straightforward. The maximum drawdown is only computed on periods for which all months are contiguous.

The figures show that the profitability of the proposed portfolios is robust with respect to the choice of the period: the annualized performance over the four decades is at least equivalent to 11.9% and can reach up to 22.6%. The volatilities lie in a window of 14%-20%. The most turbulent period is the last one because of the subprime crisis, which yielded a maximum drawdown of 55% in less than two years. In terms of relative risk-adjusted performance, the high (and positive) information ratios confirm the added value of the tactical views. Lastly, apart from the first decade, the hit ratios are all well above 50%. The overall outperformance with respect to the EW benchmark, combined with a low hit ratio (47.6%), suggests that, over this decade, the magnitude of the underperformance is much lower than the magnitude of outperformance.

The study of conditional performance in times of bull and bear markets also highlights the economic gains brought by the active views. Both in months or years of bull and bear markets, the intensity allocation outperforms the CW and the strategic benchmarks, on average. The hit ratios are all above 54% and they are much higher in bullish environments.

Lastly, we focus on two special economic configurations. The first one is the tech bubble, during which cap-weighted portfolios outranked all alternative strategies. Indeed, it is the only period which implies a negative information ratio with respect to the CW benchmark. Of course, the high annual returns (20.7%) of the proposed strategy over this period strongly mitigate the significance of this negative IR. Moreover, when compared to the strategic benchmark, the intensity-based allocation reaches a very high IR of 1.25, which demonstrates that the views are highly useful since they allow for an improvement of performance by increasing the proportion of the relevant rewarded tilts (for instance, during the tech bubble, the views indicate to decrease the proportion of the mid-cap factor and increase the proportion of the momentum factor). The second period is the subprime crisis and the two positive information ratios prove that the active tilting of the portfolio again leads to superior performance, compared to the two benchmarks.

3.3.3.3. Rebalancing Frequency

The base case results rely on a monthly rebalancing which typically generates high levels of turnover. A natural question is whether or not the performance of the portfolios is affected by less frequent asset rotations. We present in Table 21 the usual indicators related to quarterly (3M), semi-annual (6M) and annual rebalancing.

Table 21 : Rebalancing frequency

	Ann. Return	Vol	SR	MaxDD	TE (CW)	IR (CW)	TE (EW)	IR (EW)	HR(EW)
Panel A: 1M									
Intensity	15.4%	17.7%	0.57	56.0%	6.53%	0.76	1.90%	0.76	57.5%
Binary (k=0.00)	15.5%	18.2%	0.56	56.7%	7.04%	0.72	2.69%	0.58	65.1%
Binary (k=0.50)	14.5%	17.3%	0.53	55.6%	6.09%	0.66	0.79%	0.66	65.1%
Panel B: 3M									
Intensity	15.1%	17.6%	0.55	55.3%	6.46%	0.71	1.85%	0.60	55.6%
Binary (k=0.00)	15.3%	18.1%	0.55	57.4%	6.94%	0.69	2.77%	0.46	59.1%
Binary (k=0.50)	14.4%	17.3%	0.52	55.9%	6.05%	0.65	0.83%	0.53	59.1%
Panel C: 6M									
Intensity	15.2%	17.6%	0.56	55.6%	6.43%	0.73	1.82%	0.66	56.9%
Binary (k=0.00)	15.2%	18.0%	0.55	57.9%	6.91%	0.68	2.70%	0.44	56.7%
Binary (k=0.50)	14.4%	17.2%	0.52	56.1%	6.04%	0.65	0.81%	0.52	56.7%
Panel D: 12M									
Intensity	14.9%	17.3%	0.55	55.3%	6.40%	0.69	1.92%	0.49	56.7%
Binary (k=0.00)	14.9%	17.7%	0.54	55.3%	6.75%	0.65	2.59%	0.35	53.6%
Binary (k=0.50)	14.3%	17.2%	0.52	55.4%	6.02%	0.63	0.81%	0.37	53.4%
EW (factors)	14.0%	17.0%	0.51	55.7%	5.87%	0.59	0.00%	-	-

The sample period is 1972-2013 and rebalancing occurs every month (Panel A), every three months (Panel B), every 6 months (Panel C) or every 12 months (Panel D). The four tilted building blocks are all weighted according to either the maximum deconcentration principle. The allocation process is driven by $w_I(F,t)$ for the intensity approach and by $w_G(F,t)$ for the binary method. The EW portfolio of all factors is equivalent to the binary method for k=1.

For the portfolios based on the binary signals, the annualized returns are strictly increasing with the rebalancing frequency (the more often the trades, the higher the performance). The pattern is the same for the intensity signals, even though the figures are not monotonic. However, the small loss in performance is a clear indication that the signals show persistence. Volatility seems also to be increasing in trading frequency, and the net effect is that Sharpe ratios are slightly higher when rebalancing takes place at the monthly frequency. Overall, these results translate to relative risk-adjusted performance since the information ratios linked to the annual rebalancing are clearly the smallest of all.

In short, these findings suggest that while the signals seem to display persistence, there is an economic cost of trading too infrequently (especially below the six month threshold).

3.3.3.4. Turnover Reduction

The transaction costs associated to monthly trading would be too high in practical implementation since a monthly rebalancing leads to one-way turnovers far beyond 100%. A simple solution is simply to switch to a quarterly rebalancing (or even a semi-annual one). However, an alternative method also allows a strong reduction in turnover, namely the shrinkage of weights towards their previous value.

First, let us recall the formal definition of turnover:

$$Turn = \frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{N}\left|w_{i,t} - w_{i,t-}\right|,$$

where $w_{i,t}$ it the time-*t* desired weight for asset *i* at rebalancing and $w_{i,t-}$ the corresponding weight in the portfolio just before rebalancing. We then introduce the following shrinkage (bold letters stand for vectors):

$$\boldsymbol{w}_t^{\alpha} = \alpha\boldsymbol{w}_t + (1-\alpha)\boldsymbol{w}_{t-1}.$$

We underline that replacing $\boldsymbol{w}_{t-1}$ by $\boldsymbol{w}_{t-}$ does not alter the results because the discrepancy between these two vectors is usually negligible compared to that between either one and $\boldsymbol{w}_t$. When $\alpha = 1$, we recover the original strategy and when $\alpha = 0$, then the weights are reset to their initial values at each rebalancing. The corresponding performance indicators are collected in Table 22.

Apart from the maximum drawdown, all indicators are decreasing in α. The reduction in tracking error is likely to be a consequence of the decline in volatility. The loss in performance is very limited (20 basis points) up to α=0.6. Below this threshold, both absolute and relative risk-adjusted performances fall significantly. Moreover, the level of turnover (41.6%) for α=0.6 is reasonable from an implementation standpoint.

Table 22 : Turnover reduction via shrinkage

	Ann. Ret.	Vol	SR	MaxDD	TE (CW)	IR (CW)	TE (EW)	IR (EW)	HR(EW)	Turn
Intensity	15.2%	17.6%	0.56	55.3%	6.46%	0.72	1.85%	0.64	56.5%	54.6%
α = 0.8	15.1%	17.6%	0.56	55.6%	6.40%	0.72	1.72%	0.64	55.4%	47.3%
α = 0.6	15.0%	17.5%	0.56	55.8%	6.31%	0.71	1.59%	0.62	55.4%	41.6%
α = 0.4	14.8%	17.4%	0.55	55.7%	6.20%	0.69	1.40%	0.56	54.0%	36.7%
α = 0.2	14.4%	17.3%	0.53	55.4%	6.09%	0.65	1.09%	0.41	51.4%	32.5%
EW (factors)	14.0%	17.0%	0.51	55.7%	5.87%	0.59	0.00%	-	-	30.4%

The whole sample period is 1972-2013 and rebalancing is executed at a quarterly frequency. The four tilted building blocks are all weighted according to the maximum deconcentration principle. The allocation process is driven by $w_I(F,t)$ (intensity approach). The initial weights are all equal (1/4 for each factor).

3.3.3.5. Strategies Based on Terciles

The base case strategies rely on signals based on the results of Section 3.2.2 for which each variable has a high or a low state (depending on its position with respect to the long-term median). A similar construction can be performed with the results of Section 3.2.3. In accordance with these results, we exclude the variables which do not lead to monotonic average returns across the terciles.

The signals $s_{v,t}^F$ are then equal to two if the variable *v* is associated to expected outperformance of factor *F* at time *t,* equal to zero if the variables is associated to expected underperformance of the factor and to one otherwise (neither outperformance nor underperformance: the values between the two terciles). We then aggregate the signals at the factor level by combining all of the relevant variables for this factor:

$$I(F,t) = \frac{1}{2n_F}\sum_{i=1}^{n_F} s_{i,t}^F,$$

where n_F is the number of relevant variables for factor *F*. By construction, the intensity *I(F,t)* lies again between 0 and 1. The allocation to the factors remains unchanged:

$$w_I(F,t) = \frac{I(F,t)}{\sum_{F=1}^{4} I(F,t)},$$

and we restrict the robustness check to the intensity approach for the sake of brevity. The performance indicators related to this strategy are displayed in Table 23.

Table 23 : Allocation based on terciles

	Ann. Return	Vol	SR	MaxDD	TE (CW)	IR (CW)	TE (EW)	IR (EW)	HR(EW)
Original Method	15.4%	17.7%	0.57	56.0%	6.53%	0.76	1.90%	0.76	57.5%
Tercile intensity	15.1%	18.0%	0.55	58.3%	6.65%	0.69	2.31%	0.48	56.3%
EW (factors)	14.0%	17.0%	0.51	55.7%	5.87%	0.59	0.00%	-	-

The whole sample period is 1972-2013 and rebalancing is executed at a monthly frequency. The four tilted building blocks are all weighted according to the maximum deconcentration principle. The allocation process is driven by $w_I(F,t)$ (intensity approach).

A conclusion that can be drawn from these results is that using terciles does not improve the performance of factor timing strategies on any level: annual returns are lower and risk (volatility and maximum drawdown) is higher. Moreover, the tracking errors with respect to both benchmarks are higher, which implies a strong decrease in information ratios. As such, even though this approach still leads to outperforming the two benchmarks, it appears sub-optimal compared to the original strategy based on a parsimonious classification of market conditions in two states.

4. Introducing a Dynamic Management of Tracking Error Risk

In a context where an actively managed allocation to smart factor indices is regarded as a reliable cost-efficient substitute to active portfolio management based on security selection, one important challenge is to maintain a controlled level of tracking error with respect to the cap-weighted index.

One first approach to asset allocation decisions in the presence of tracking error constraints consists in explicitly introducing the ex-ante tracking error budget in the optimization procedure that defines the tactical allocation benchmark. This approach, however, requires an ad-hoc adjustment to the active allocation process which is detrimental to the performance of this process.

Another competing approach, known as the *core-satellite approach* in investment practice, has received increasing attention from investors who are trying to structure their portfolio management process in a coherent manner. This portfolio management technique divides the portfolio into a core component, which replicates the investor's specifically designed benchmark (here, the cap-weighted index), and an actively managed component, here the strategic and/or tactical allocation to smart factor indices. According to this approach, the tracking error budget is simply managed in terms of one single degree of freedom, namely the allocation to the core versus the satellite portfolio. Mechanically, large weights in the core component will lead to low levels of tracking error, but also to higher opportunity costs because the smart-weighted satellite is expected to deliver higher returns. Consequently, an efficient mix between the core and the satellite requires a fine-tuning of the trade-off between relative risk and absolute performance. One important limitation of the core-satellite approach is that it treats equally downside and upside risk relative to the cap-weighted benchmark. This is a concern in a situation where most of the relative risk of smart factor index portfolios is on the upside with respect to the cap-weighted index. In this end, a purely symmetric management of tracing error budgets often leads asset managers to be extremely conservative in their active risk exposure, with an associated prohibitive opportunity cost.

While core-satellite investments can be implemented in a static manner, such that the proportion invested in the core and the satellite remains constant over time, the full potential of this portfolio management technique is obtained when dynamically adjusting the allocation between the satellite and the core portfolios over time. Based on the work of Amenc, Malaise, and Martellini (2004), this approach leads to an increase in the fraction allocated to the satellite when the satellite has outperformed the benchmark. The focus of this Section is a comprehensive examination of the benefits of relative risk control of the strategically and/or tactically managed portfolio of smart factor indices through suitable adaptations of the basic dynamic core-portfolio strategy introduced in Amenc, Malaise, and Martellini (2004). Several forms of dynamic risk budgeting techniques dedicated to providing a risk-controlled exposure to smart factor indices will be empirically examined, including the use of various reset/ratchet strategies and the use of floor levels based on relative drawdown, which have been shown to alleviate the concern over the excessive path-dependency of standard forms of dynamic core-satellite strategies. We will also analyze the benefits of the use of time-varying multipliers that will taken as function of current market conditions, which is an effective and parsimonious way to combine the benefits of active allocation and risk control.

Finally, we also provide an explicit measure of the reduction in the opportunity cost related to the management of the tracking error risk budget allowed by the use of dynamic dissymmetric risk management versus standard static symmetric core-satellite investment decisions. This reduction opportunity cost will be formally measured by the difference in outperformance when tracking error constraints are managed via a static approach with respect to the situation when they are managed via a dynamic approach.

4.1. Data and Monte-Carlo Protocol

The data for this whole Section consists of two time-series of total returns: those of the (core) cap-weighted index (here the universe corresponds to that of the S&P 500 index) and those of the (satellite) MBMS R-ERC index of the Scientific Beta platform. As in the previous Section, the sample period begins in January 1972 and ends in December 2013.

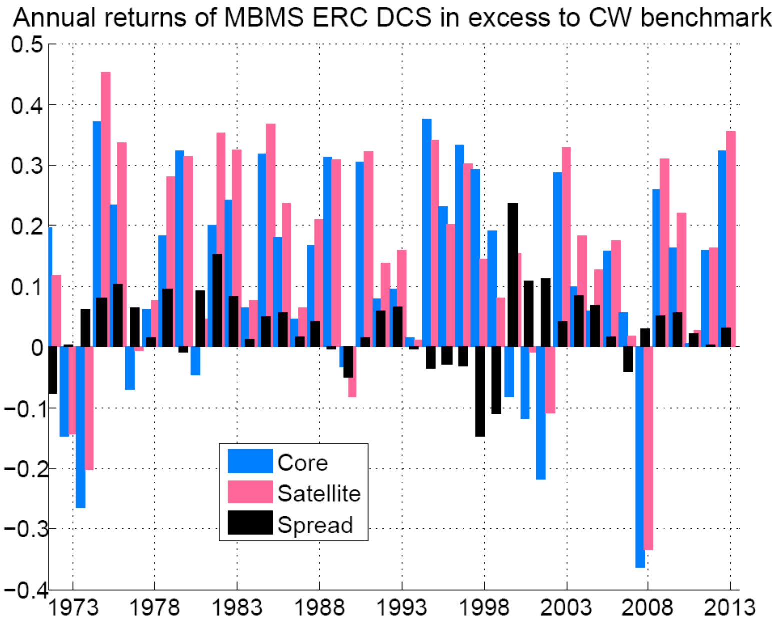
Based on this data, we will additionally perform Monte-Carlo simulations (30,000 trajectories of 30 years of monthly returns). The monthly returns are modeled according to a bivariate Gaussian distribution with annualized parameters estimated over the full sample period and equal to:

$$\boldsymbol{\mu} = \begin{bmatrix} 0.1188 \\ 0.1539 \end{bmatrix}, \qquad \boldsymbol{\Sigma} = \begin{bmatrix} 0.0252 & 0.0228 \\ 0.0228 & 0.0240 \end{bmatrix}, \quad \rho = 0.92.$$

The smart-weighted index offers both a higher annualized return and a lower volatility (15.5% versus 15.9%). The annualized tracking error (computed on monthly returns) between the two indices is equal to 5.04%. The information ratio of the satellite with respect to the core is equal to 0.70 and the relative maximum drawdown (MDD of the ratio of the prices of the indices) of the satellite with respect to the core reaches 26.6%. Figure 2 displays the calendar annual returns of the two indices, as well as the difference between the two.

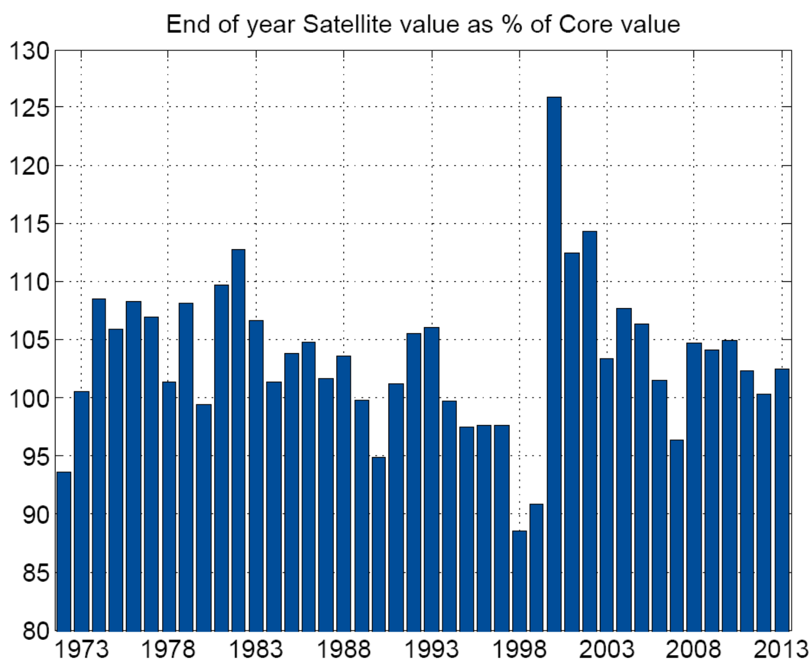
While the smart weighted portfolio shows a substantial level of outperformance compared to the CW portfolio for most years, it does happen that the CW portfolio exhibits a stronger performance in some market conditions. For further clarity, Figure 3 shows the annual performance of the satellite as a percentage of the core.

Figure 2: Annual returns of core and satellite



This bar plot displays the calendar annual returns of the core (CW index of all stocks of the S&P 500 universe) and the satellite (Scientific Beta MBMS R-ERC index). The rebalancing of the indices occurs every quarter (in March, June, September and December).

Figure 3: Relative annual performance of the satellite with respect to the core



This bar plot displays the annual performance of the satellite as measured by percentage of the core. More precisely, we plot $\mathrm{BAR}_i = \frac{S(i+\Delta t)}{S(i)} \times \frac{C(i)}{C(i+\Delta t)}$, where *i* stands for the beginning of each year and Δt the duration of the year.

Over the 42 years of the sample, 4 years end up below 95%, which underlines the potential for relative underperformance of smart weighted satellite portfolio with respect to the CW core portfolio in specific market conditions, in particular during the 1997-2000 tech bubble, where the CW index performance was favored by the momentum driven nature of the market.

It is precisely the goal of the dynamic core-satellite strategy to generate a high access to the relative upside potential of the smart weighted index, while maintaining the maximum underperformance at a limited level. In its simplest version, the Dynamic Core-Satellite (DCS) approach stipulates that the allocation to the satellite be equal to a multiple of the difference between the value of the portfolio and a specified floor, which is usually taken to be a fraction of the core. More precisely, at each rebalancing date, the proportion invested in the satellite is equal to:

$$w(t) = \min\left(1, \max\left(0, m\frac{P(t) - F(t)}{P(t)}\right)\right),$$

where *P(t)* is the time-*t* value of the portfolio and *F(t)* is the value of the floor. In the base case, the floor is a fixed multiple of the core value: $F(t) = kC(t)$. Both the values of the core *C(t)* and the value of the satellite *S(t)* are normalized so that *C(0)=S(0)*=1. The rebalancing takes place on the first trading day of each month. As the portfolio is fully invested, the proportion allocated to the core is given by 1-*w(t)*. The initial weight of the satellite is *m(1-k)*.

Unfortunately, this original version of DCS has one major drawback, namely its path-dependency: depending on its starting point, if the core starts by outperforming the satellite, then one may end up hitting the floor quickly and then be left certain to underperform the core at terminal date with a terminal wealth equivalent to *k* times the value of the core. In the next subsections, we propose various approaches that can be used to alleviate this concern.

4.2. Dynamic Core-Satellite with Annual Reset

As mentioned above, one of the main drawbacks of the DCS approach is that once it is fully invested in the core (in which case *P(t)=kC(t)*), the allocation to the satellite remains null until expiry. While this is no problem when the satellite displays continued excess performance with respect to the core, it can lead to notoriously disappointing results when the core outperforms the satellite soon after inception, in which case it might happen that the investor gets stuck at a wealth level equal to *k* times the core until maturity.

A simple solution to this problem is to reset the weights to their original value (*w(t)=m(1-k)*) at pre-specified fixed or random dates. In this case, the floor must be updated at the date of the reset:

$$F(t+s) = k\frac{P(t)}{C(t)}C(t+s), \quad s > 0.$$

The reset date can be fixed arbitrarily (for instance the first day of each year), or depending on the relative performance of the portfolio with respect to the core (we will develop this idea in the next subsection).

Given the purposes of DCS strategies, we must now define new performance indicators which will assess whether or not these strategies achieve their objectives. In the case of DCS with annual reset, we will compute:

- The minimum proportion of core secured annually, defined as

$$PCS = \min_i \frac{P(i+\Delta t)/P(i)}{C(i+\Delta t)/C(i)},$$

  where *i* denotes the beginning of each year and Δ*t* corresponds to the length of 1 year.

- The probability for the portfolio to reach 95% of the value of the core after 1 year of investment:

$$P95 = \frac{\mathrm{Card}\left(i, \frac{P(i+\Delta t)}{P(i)} > 0.95 \times \frac{C(i+\Delta t)}{C(i)}\right)}{\mathrm{Card}(i)},$$

  where Card (the cardinal) is the operator that counts the number of occurrences for which the condition is satisfied and Card(*i*) equals the number of samples.
- The average annual outperformance of the core with respect to the satellite:

$$AAO = \frac{1}{N}\sum_{i=1}^{N}\left[\frac{P(i+\Delta t)}{P(i)} - \frac{C(i+\Delta t)}{C(i)}\right].$$

- The probability of one year outperformance:

$$P1YO = \frac{\mathrm{Card}\left(i, \frac{P(i+\Delta t)}{P(i)} > \frac{C(i+\Delta t)}{C(i)}\right)}{\mathrm{Card}(i)},$$

  and in this case, the *i* span all available dates, while for the previous indicators, *i* stood for the beginning of non-overlapping one year periods.

The values of these indicators are gathered in Table 24.

The choice of *k*=0.95 implies that we aim at protecting 95% of the value of the core. In almost all cases, this is satisfied, even if we observe a very limited amount of gap risk, that is the risk that the portfolio value falls below the floor between two rebalancing dates, for high values of the multiplier. Across all Monte-Carlo simulations, it appears that the maximum *static* allocation to the smart weighted satellite versus the CW core that allows an investor to obtain a 100% probability of reaching 95% of the core every year (any higher proportion of the satellite decreases this probability below 100%) is a mere 24%. The comparison of average performance highlights the opportunity cost of this static core-satellite strategy: the 87 basis points outperformance generated is 30 basis points lower than that of the DCS strategy with *m*=6 which satisfies the same condition, namely the respect of the floor with a 100% probability.

Table 24 : DCS strategies with annual reset (*k*=0.95)

	Min prop. of core secured	Prob. to reach 95% of core	Average annual outperformance	Prob. of 1Y outperf.
Panel A: Monte-Carlo simulations				
Annual reset, *m*=2	98.07%	100.00%	0.37%	70.28%
Annual reset, *m*=4	96.74%	100.00%	0.76%	69.10%
Annual reset, *m*=6	95.77%	100.00%	1.17%	67.90%
Annual reset, *m*=8	94.89%	99.99%	1.59%	66.33%
Annual reset, *m*=10	93.61%	99.95%	2.01%	65.50%
FM 24% Satellite	95.06%	100.00%	0.87%	71.28%
Panel B: Historical sample (1972-2013)				
Annual reset, *m*=2	98.90%	100.00%	0.35%	72.94%
Annual reset, *m*=4	97.98%	100.00%	0.73%	72.65%
Annual reset, *m*=6	97.21%	100.00%	1.15%	72.36%
Annual reset, *m*=8	96.55%	100.00%	1.61%	72.00%
Annual reset, *m*=10	95.90%	100.00%	2.10%	71.61%
FM 24% Satellite	97.15%	100.00%	0.78%	73.29%

Each year, we compute the proportion of core secured and then look at the minimum over all years of simulated data (Panel A) or all 42 years in the historical sample (Panel B). The probability of reaching 95% is calculated as the number of years for which the final value (over one year) of the portfolio is higher than 95% of the value of the core, divided by the total number of years. The outperformance is computed as the average annual return over all years. The probability of outperformance is computed as the number of 1Y windows for which the strategy has outperformed the core, divided by the total number of available 1Y windows.

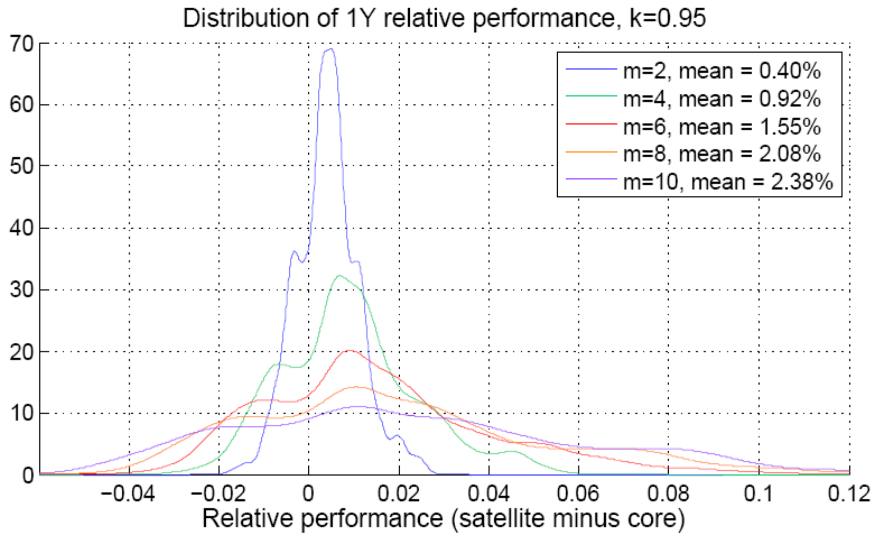
That dynamic strategies have a much higher upside compared to the static strategy that does not violate the floor (here the static 24%/76% strategy) result confirms the general finding that a non-linear exposure to the underlying source of performance through insurance-strategy is a less expensive approach compared to excessive levels of static and symmetric hedging when it comes to protecting short-term risk budgets.

The conclusions for the historical sample are the same: an investor seeking to protect a portfolio against a relative underperformance is better off with a DCS strategy with annual reset compared to the static core-satellite strategy because it delivers higher average performance while controlling downside risk relative to the benchmark.

Surprisingly, the probability of annual outperformance is decreasing with *m*. Given the increasing pattern of *m* with annual performance, this can only mean that the magnitude of positive returns is much larger when *m* is large. This intuition is confirmed by

Figure 4, which plots the empirical density of relative annual returns (over all available one year windows in the Monte-Carlo simulation).

Figure 4: Empirical distribution of excess returns with respect to the core



This plot shows the fitted probability density function of the returns of the DCS strategies, in excess of those of the core. The returns are computed over all available windows of one year simulated in the Monte-Carlo protocol.

The main conclusion is that as *m* increases, the tails (both left and right) of the distribution become heavier. As such, even in the DCS strategy, the investor must be willing to take more risk to earn more *on average*. A natural extension of the annual reset is to change the frequency of the resetting. Since mandates have maturities that are longer than one year, we test our methodology when the reset takes place every three years. The corresponding results are gathered in Table 25, Panel A (Monte-Carlo simulations) and Panel B (Historical sample).

The results are even more in favor of the DCS strategy with reset compared to the fixed-mix alternative, which can be explained that the investor has more time to benefit from the outperformance potential of the smart-weighted portfolio before a reset decision takes place. In the Monte-Carlo experiment, the 18% dilution can be compared to the case m=8 and the spread in average performance is equal to 125 basis points. In the historical sample, the increase in performance can reach 216 basis points because the strategy with *m*=10 does not violate the floor. This test of robustness further highlights the benefits of the resetting procedure.

Table 25 : Strategies with triennial reset (*k*=0.95)

	Min prop of core secured	Prob to reach 95% of core	Average 1Y Outperf.	Proba of 1Y outperf.
Panel A: Monte Carlo simulations				
Triannual reset m=2	97.65%	100.00%	0.41%	69.73%
Triannual reset m=4	96.39%	100.00%	0.88%	67.76%
Triannual reset m=6	95.60%	100.00%	1.42%	65.88%
Triannual reset m=8	95.09%	100.00%	1.92%	64.23%
Triannual reset m=10	93.98%	99.84%	2.28%	63.37%
FM 18% Satellite	95.01%	100.00%	0.67%	71.37%
Panel B: Historical sample (1972-2013)				
Triannual reset m=2	98.54%	100.00%	0.46%	72.81%
Triannual reset m=4	97.45%	100.00%	1.06%	72.21%
Triannual reset m=6	96.63%	100.00%	1.81%	71.61%
Triannual reset m=8	96.05%	100.00%	2.45%	70.94%
Triannual reset m=10	95.63%	100.00%	2.84%	70.45%
FM 18% Satellite	97.86%	100.00%	0.65%	73.32%

Each year, we compute the proportion of core secured over 3 years and then look at the minimum over all years of simulated data (Panel A) or all 42 years in the historical sample (Panel B). The probability of reaching 95% is calculated as the number of years for which the final value (over three years) of the portfolio is higher than 95% of the value of the core, divided by the total number of years. The outperformance is computed as the average annual return over all years. The probability of outperformance is computed as the number of 1Y windows for which the strategy has outperformed the core, divided by the total number of available 1Y windows.

4.3. Dynamic Core-Satellite with Random Ratchet Principle

Since the purpose of the core-satellite approach is to secure at least *k* times the value of the core, an intuitive extension is to reset the weights of the strategy when the portfolio reaches (1+*i*(1-k)*)/*k* times the value of the core, for *i*=0,1,… This ratchet effect would mechanically increase the floor to (1+*i*(1-k)*) times its original value. For instance, if we set *k*=0.9, this would allow to secure successively 100%, 110%, 120%,… of the core.

To properly evaluate the properties of the ratchet, we introduce two new indicators:

- The probability to reach a floor larger or equal than 100% of the core, given by:

$$\frac{\text{Card}\left(i, \frac{P(T)}{P(1)} \geq \frac{C(T)}{C(1)}\right)}{\text{Card}(i)},$$

  where T is the final date (maturity) of the strategy.

- The proportion of time spent with a floor larger or equal to 100% of the core, given by:

$$\frac{1}{N}\sum_{i=1}^{N}\frac{T-\tau_i}{T},$$

where τ_i is the time spent for the ratcheting strategy to increase the floor to 100% of the core (expressed in years).

Naturally, both indicators strongly depend on the maturity of the strategy. Consequently, we will test five different values for T: 3, 5, 10, 20 and 30 years.

Table 26 : DCS strategies with ratchet (*k*=0.9)

Panel A: Prob floor>=100%	3Y	5Y	10Y	20Y	30Y
Ratchet m=4	25.41%	49.73%	80.39%	95.72%	98.92%
Ratchet m=6	45.19%	65.67%	85.88%	96.26%	98.71%
Ratchet m=8	55.99%	72.38%	87.26%	95.83%	98.09%
Panel B: %Time spent with floor above core					
Ratchet m=4	5.25%	18.65%	43.71%	66.99%	77.00%
Ratchet m=6	17.05%	33.52%	55.82%	74.10%	81.83%
Ratchet m=8	24.98%	41.72%	61.35%	76.95%	83.53%
Panel C: Average 1Y Outperformance					
Ratchet m=4	1.63%	1.65%	1.66%	1.65%	1.65%
Ratchet m=6	2.34%	2.32%	2.26%	2.21%	2.19%
Ratchet m=8	2.86%	2.78%	2.64%	2.52%	2.46%
Panel D: Prob. 1Y Outperf.					
Ratchet m=4	68.00%	68.39%	68.37%	68.42%	68.44%
Ratchet m=6	66.74%	67.02%	66.85%	66.89%	66.88%
Ratchet m=8	66.34%	66.52%	66.13%	65.98%	65.91%

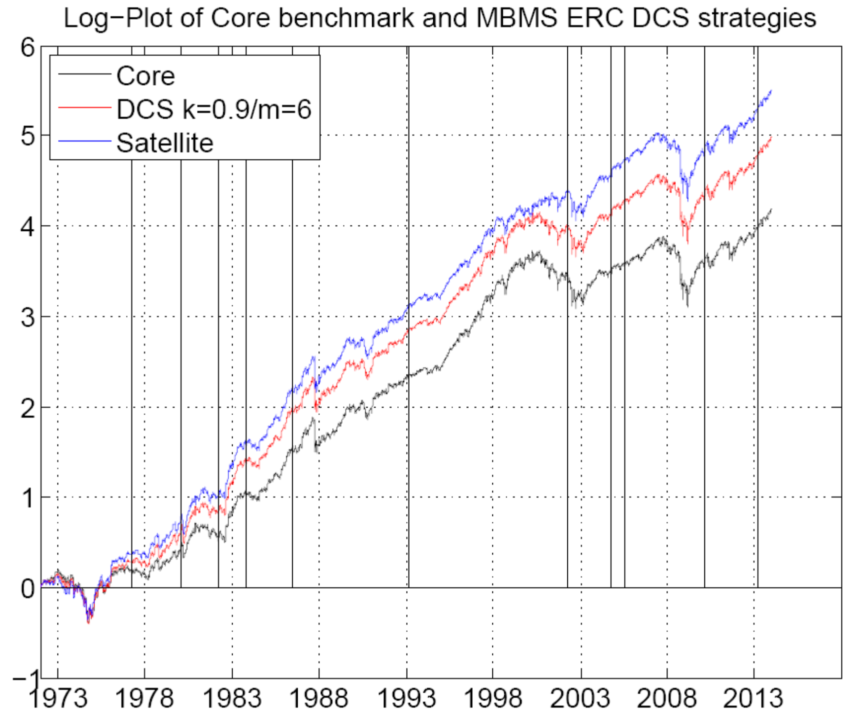
The probability to reach a floor of 100% of the core is computed as the number of paths which satisfy this assumption at maturity, divided by the total number of paths (10,000). The proportion of time spent with a floor larger or equal to the core is computed as the average of the number of months with F≥C divided by the total number of months. The outperformance is calculated as the average (across all simulations) of the geometric returns over the whole trajectories (ie. 3, 5, 10, 20 or 30 years). The probability of outperformance is computed as the number of 1Y windows for which the strategy has outperformed the core, divided by the total number of available 1Y windows.

We see that the probability to reach a floor equal or larger than the core and the time spent when this is verified is strongly increasing with the time to maturity. This is again because longer maturities give more time to the satellite to outperform the core. The figures also increase with *m* for the same reasons because a higher *m* implies a higher proportion invested in the satellite. Similarly to the other strategies, the average returns increase with *m*, but the probability of one year outperformance slightly decreases with *m*.

In addition to the results of the Monte-Carlo framework, we provide in Figure 5 the log-plot of the values of the portfolio resulting from the ratchet strategy for *k*=0.9 and *m*=6.

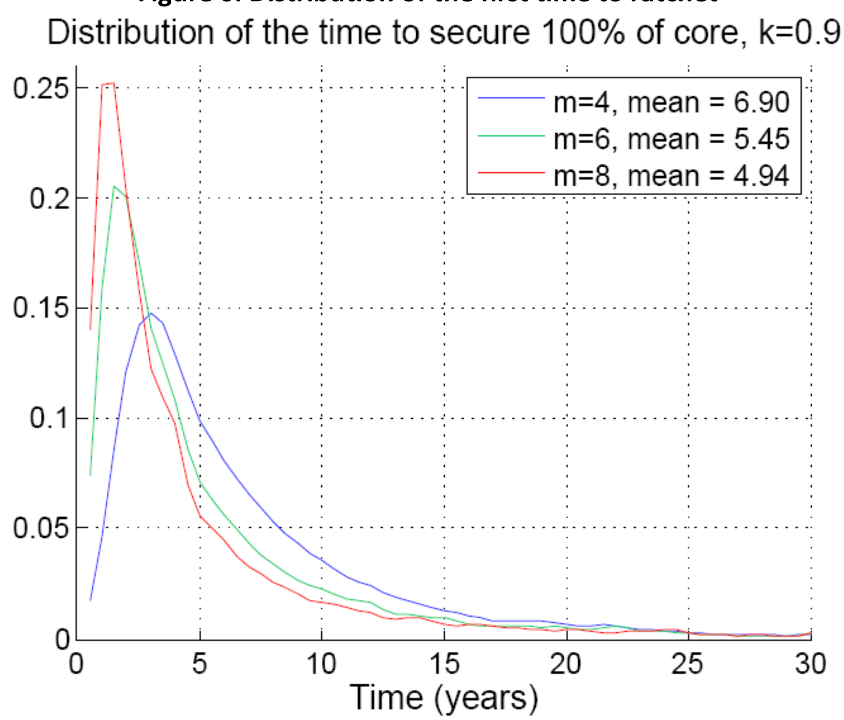
The graph shows the various lengths of time that it can take between two ratchets (between one and nine years). In this sample, it takes 5.2 years before the first ratchet and then 2.8 years to reach the second one. In the end, the floor is ratcheted 11 times which means that the strategy eventually allo<ws an investor to secure 200% of the value of the core. Lastly, to further document the properties of the ratchet mechanism, we have plotted the empirical distribution of the first ratchet time (in years) in Figure 6.

Figure 5: Log-plot of the DCS strategy with ratchet (*k*=0.9, *m*=6)



This figure displays the log of the values of the core, the satellite and the DCS strategy. All time-series are normalized to one at inception (January 1972). The vertical lines show the moments when ratcheting takes place.

Figure 6: Distribution of the first time to ratchet



This plot shows the fitted probability density function of the time required before the first ratchet. The sample consists of the 10,000 Monte-Carlo simulations.

The graph shows that a higher multiplier shifts the distribution to the left and thus reduces the expected time before the first ratchet. This was expected because when *m* is large, a higher proportion of the portfolio is invested in the satellite which increases the upside potential of the strategy and it thus takes less time to reach the levels at which the ratchet effect is activated.

4.4. Introducing Relative Max Drawdown Constraints

In principle, the DCS technique relies on the assumption that the satellite will outperform the core in the long run. Inevitably, periods of the opposite situation will occur which will entail negative relative performance. For instance, our Monte-Carlo simulations have shown that over 10,000 trajectories of 30 years, the maximum relative drawdown (computed as the maximum drawdown of the ratio of the value of the portfolio divided by the value of the core) can reach 40% for the DCS strategy with annual reset when *m*=6 and *k*=0.95. As such, an annual protection against a 5% relative underperformance, when the core durably outperforms the satellite over several consecutive years, can lead to a much worse compounded underperformance.

In order to avoid this risk of sustained relative underperformance, we propose an alternative definition of the floor which enables to cap the risk of relative maximum drawdown. More precisely, we track the maximum of the portfolio/core ratio:

$$M(t) = \max_{s \le t} \frac{P(s)}{C(s)}, \qquad M(0) = 1.$$

In times when, as expected, the satellite outperforms the core, the *P(t)/C(t)* ratio increases above 1. However, when the reverse scenario happens, then the ratio decreases. In order to prevent a significant loss in the relative outperformance progressively accumulated, we define the following floor:

$$F(t) = kM(t)C(t).$$

Consequently, notwithstanding gap risk stemming from discrete monitoring, we expect the DCS strategy to satisfy:

$$\frac{P(t)}{C(t)} \ge kM(t),$$

so that the ratio *P(t)/C(t)* is protected against any drawdown of magnitude *k* or more. Under these assumptions, the proportion invested in the satellite is bounded above by *m(1-k)* at all rebalancing dates.

The results corresponding to this strategy are detailed in Table 27 for *k*=0.9.

Table 27 : DCS with maximum relative drawdown protection (*k*=0.9)

	Maximum Rel. DD. / Core		Outperf. (geom)		Prob. 1Y outperf.	
	Historical (42Y)	Monte Carlo (30Y)	Historical (42Y)	Monte Carlo (30Y)	Historical (42Y)	Monte Carlo (30Y)
MRDD *m*=2	5.06%	7.50%	0.68%	0.69%	72.98%	70.01%
MRDD *m*=4	7.85%	9.50%	1.26%	1.25%	72.70%	68.54%
MRDD *m*=6	9.31%	9.94%	1.73%	1.66%	72.34%	66.99%
MRDD *m*=8	10.03%	10.21%	2.06%	1.91%	71.93%	65.27%
MRDD *m*=10	10.37%	12.14%	2.25%	1.99%	71.22%	63.32%
FM 16% Satellite	5.19%	9.95%	0.57%	0.60%	73.33%	71.37%

The maximum relative drawdown is computed as the maximum value of M(t) – the maximum relative drawdown – over all 10,000 simulations. The outperformance is computed as the difference in geometric returns over the whole period. The probability of outperformance is computed as the number of 1Y windows for which the strategy has outperformed the core, divided by the total number of available 1Y windows.

First, we see that gap risk materializes above *m*=6. We can thus compare this case with the fixed-mix portfolios which satisfy the MDD constraint either on historical data or on simulated data. The increase in returns is at least 50 basis points and can reach 116 basis points with the 16% dilution on the historical sample. We highlight, in passing, the strong similarity in returns between the two samples.

The probability of outperformance is not much impacted by *m* in the historical sample, while it decreases by 7% when switching from *m*=2 to *m*=10. The values for the fixed-mixes are also hardly distinguishable, for both samples.

4.5. Benefits of Time-Varying Multipliers

The multiplier is a key parameter which describes how aggressively a given risk budget (here a relative risk budget) is spent by the investor. Intuition suggests that risk budgets should be spend less aggressively in periods of higher uncertainty, and they should be spent more aggressively when market conditions suggest that the smart-weighted satellite portfolio is expected to out-perform more strongly the cap-weighted reference. In this Section, we will use the insights obtained in the Section 3 regarding the evidence of predictability in the relative performance of smart-weighted portfolios with respect to cap-weighted portfolios so as to generate additional benefits in terms of out-performance potential of dynamic core-satellite strategies. The main idea is to adapt the multiplier, *m*, to conditional spreads in performance between the core and the satellite. For instance, if the satellite is expected to particularly outperform the core in a particular regime, then *m* can be increased in such periods to further take advantage of the discrepancies in returns between the core and the satellite. In the end, integrating a tactical factor allocation process within the dynamic core-satellite framework allows investor to better manage the risks related to uncertainty about active views. In other words, in addition to tactical smart factor bets within the actively managed smart beta portfolio, it is conceivable to introduce tactical views on the global performance of the smart-weighted versus cap-weighting portfolios through a time-varying multiplier strategy.

Because the spread in returns between the CW index and the smart-weighted portfolio typically shows a positive exposure to the low volatility and momentum factors and a negative exposure to value and size (see Amenc et al. (2014)), we construct the following indicator so that is meant to signal expected outperformance of the satellite with respect to the core:

$$I^*(t) = \frac{I(MCAP, t) + I(VAL, t) - I(MOM, t) - I(LVOL, t)}{2}.$$

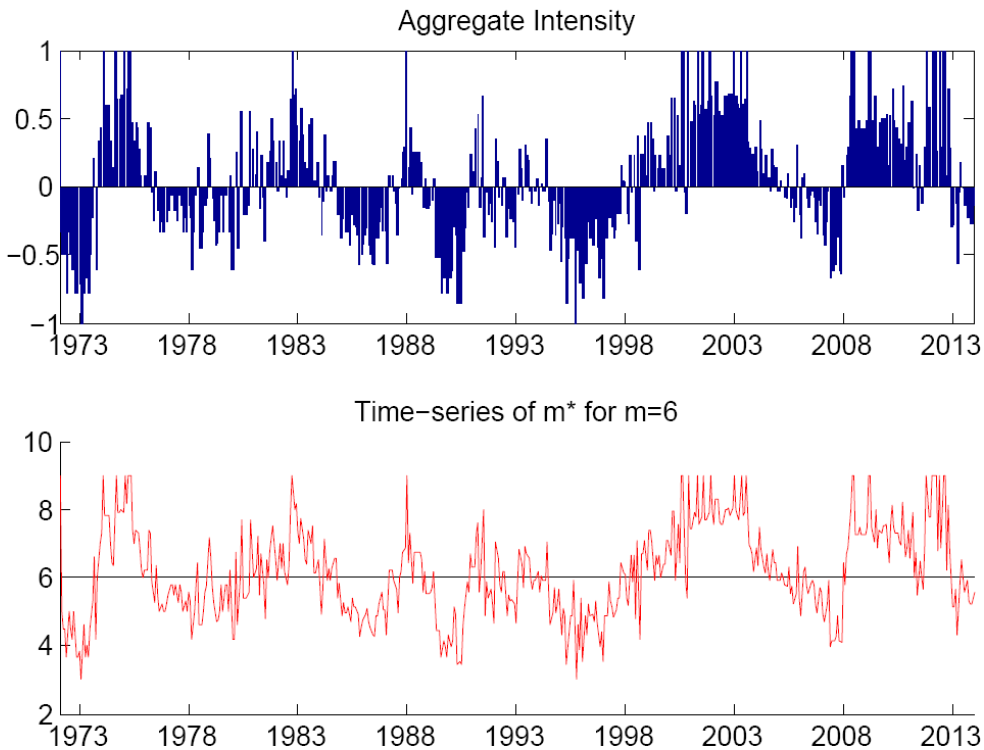
Here the factors are indexed straightforwardly by MCAP (mid cap), VAL (value), MOM (winners) and LVOL (low volatility).

By construction, *I*(t)* lies between -1 (favourable to the core) and +1 (favourable to the satellite). The multiplier is then computed as a simple function of this aggregated intensity:

$$m^*(t) = m \times \frac{I^*(t) + 2}{2},$$

which implies that *m*(t)* ranges between *m*/2 and 3*m*/2. We provide the time-series of both the intensity *I*(t)* and *m*(t)* in Figure 7 below.

Figure 7: Time-series of aggregate intensity and varying multiplier (*m*=6)



This figure plot the time-series of the aggregate intensity *I*(t)* (upper graph) and the corresponding time-varying multiplier for a base case value of *m*=6.

We start by verifying that the constructed indicator has predictive power over the conditional performance of the core versus the satellite portfolios. To this purpose, we split the sample into four subgroups, depending on the value of *I*(t)* and we then compute the average return of both the core and the satellite. The corresponding values are collected in Table 28.

Table 28 : Conditional performance of core and satellite

I*(t)	Core	Satellite	Sat-Core
[-1,-0.5)	1.72%	1.57%	-0.16%
(-0.5,0)	0.95%	1.16%	0.21%
(0,0.5)	0.65%	0.89%	0.24%
(0.5,1]	1.03%	1.71%	0.68%

This table displays the average monthly returns of the core and satellite when conditioned by the level of *I*(t)*. The third column is simply the difference between the two figures. The sample starts in January 1972 and ends in December 2013.

Table 28 confirms that *I*(t)* can indeed be used as a predictor of relative performance between the core and the satellite. The relative returns are strictly increasing with the values of *I*(t)*, as expected. *I*(t)* correctly predicts the sign of outperformance in 54.4% of the months in the sample period, and the sign of outperformance *net of average outperformance* (the most relevant quantity) 56.9% of the time. Superior results could arguably be obtained by a model specifically dedicated to active timing decisions between CW and SW, as opposed to simply focusing on factor performance, but this is beyond the scope of this analysis. As such, the time-varying multiplier *m*(t)* is expected to further add values to all aforementioned DCS strategies.

We quantify the added value of *m*(t)* on the historical sample in the following three tables. We start with the DCS with annual reset in Table 29.

Table 29 : DCS with annual reset (*k*=0.95) and time-varying multiplier

	Min prop of core secured	Prob to reach 95% of core	Average 1Y Outperf
Annual reset m=4	97.98%	100.00%	0.73%
Annual reset m=6	97.21%	100.00%	1.15%
Annual reset m=8	96.55%	100.00%	1.61%
Annual reset *m*(t), m=4*	97.87%	100.00%	0.85%
Annual reset *m*(t), m=6*	96.96%	100.00%	1.35%
Annual reset *m*(t), m=8*	96.18%	100.00%	1.90%

Each year, we compute the proportion of core secured and then look at the minimum over all 42 years of historical data. The probability of reaching 95% is calculated as the number of years for which the final value (over one year) of the portfolio is higher than 95% of the value of the core, divided by the total number of years (42). The outperformance is computed as the average annual return over all years.

The time-varying multiplier does not alter the basic purpose of the strategy, which is to secure 95% of the core every year. However, it increases the average performance by 15 bps (*m*=4) to 30 bps (*m*=8). Accordingly, it appears that a DCS strategy would benefit from the use of a time-varying multiplier. These gains also materialize for the DCS with ratchet, as we see in the following table.

Table 30 : DCS with ratchet (*k*=0.9) and time-varying multiplier

	Prob floor>=100%	Time to reach floor>=100%	Av. Annual Rel. Perf.
Ratchet m=4	100.00%	5.92 Y	1.60%
Ratchet m=6	100.00%	5.24 Y	2.13%
Ratchet m=8	100.00%	5.23 Y	2.43%
Ratchet *m*(t), m*=4	100.00%	5.06 Y	1.85%
Ratchet *m*(t)*, *m*=6	100.00%	4.14 Y	2.43%
Ratchet *m*(t)*, *m*=8	100.00%	4.12 Y	2.84%

The probability to reach a floor of 100% of the core is equal to one because in all cases, the strategy manages to ratchet at least once before the end of the 42 years. The time it takes to reach the first ratchet is given in the second column. The outperformance is calculated as the geometric return over the whole sample.

Applying the time-varying multiplier to the strategy with ratchet generates an increase of 25 basis points (*m*=4) to 40 basis points (*m*=8) in terms of excess performance, compared to the base case. As shown in Figure 5, the ratchet effect occurs 11 times for *m*=6. This number increases to 13 when the time-varying multiplier is enforced and to 17 if *m*=8 (compared to 13 with constant *m*).

Lastly, we document the gains provided by the time-varying multiplier on maximum drawdown driven DCS strategies in Table 31.

Table 31 : DCS with maximum relative drawdown (k=0.9) and time-varying multiplier

	Maximum Rel. Drawdown / Core	Outperf. (geom)
MRDD *m*=4	7.85%	1.26%
MRDD *m*=6	9.31%	1.73%
MRDD *m*=8	10.03%	2.06%
MRDD *m*(t), m*=4	8.00%	1.48%
MRDD *m*(t)*, *m*=6	9.42%	2.02%
MRDD *m*(t), m*=8	10.08%	2.42%

The figures correspond to the values over the whole historical data (42 years).

Similarly to the improvements achieved with the other strategies, we acknowledge an increase in performance between 22 basis points (*m*=4) and 36 basis points (*m*=8). At the same time, the maximum drawdown levels are only mildly impacted. Overall, the benefits of the time-varying multiplier can be summarized as follows: the strategies provide the same protection but generate higher returns on average. The gains lie between 20 and 40 basis points per year.

5. Conclusion: Multi Smart Beta Allocation as an Opportunity for Active Asset managers

While the rapid emergence of smart beta benchmarks is often perceived as a threat for traditional active managers, who face increasing competition from these cost-efficient investment vehicles with a robust ability to outperform cap-weighted indices, they can also be regarded as an opportunity. Smart factor indices with controlled and well-rewarded factor tilts can indeed be used as convenient building blocks in active factor allocation exercises.

Given that most of the abnormal performance of active managers supposedly involved in security selection comes from factor timing skills, it can be expected that utilizing smart factor indices, as opposed to individual securities, will lead to substantial efficiency gains. This paper can be seen as an attempt to showcase the benefits that active managers and asset owners can expect from dynamically allocating to smart factor indices, with a focus on efficiently reacting to changes in market conditions, as well as efficiently spending relative risk budgets with respect to a cap-weighted reference portfolio.

Given the current interest in multi-asset investment solutions, a natural extension of this work would consist of exploring the benefits of active allocation decisions beyond the equity asset class, and analyze the conditional performance of risk factors that could explain the returns on bonds and commodities in particular, in addition to equities. Specific challenges that would need to be addressed in this effort are the relative lack of maturity of smart beta indices in the fixed-income space, as well as the potential difficulty in timing risk factors across asset classes. We leave these interesting questions for further research.

References

- Aarts, F., and T. Lehnert (2005), On style momentum strategies, *Applied Economics Letters*, 12(13), 795-799.
- Acharya, V. V., and L.H. Pedersen (2005), Asset pricing with liquidity risk, *Journal of Financial Economics*, 77(2), 375-410.
- Amenc, N., F. Goltz, A. Lodh and L. Martellini (2012), Diversifying the Diversifiers and Tracking the Tracking Error: Outperforming Cap-Weighted Indices with Limited Risk of Underperformance, *Journal of Portfolio Management,* 38 (3), 72-88.
- Amenc, N., F. Goltz and L. Martellini (2013), Smart Beta 2.0, *EDHEC-Risk Institute Position Paper*.
- Amenc, N., F. Goltz, A. Lodh and L. Martellini (2014), Towards Smart Equity Factor Indices: Harvesting Risk Premia Without Taking Unrewarded Risks, *Journal of Portfolio Management*, forthcoming.
- Amenc, N., P. Malaise and L. Martellini (2004), Revisiting core-satellite investing - A dynamic model of relative risk management, *Journal of Portfolio Management*, 31 (1), 64-75.
- Amenc, N., P. Malaise, L. Martellini, and D. Sfeir (2004), Portable Alpha and Beta Strategies in the Euro Zone, *Journal of Portfolio Management*, 30(4), 204-215.
- Amihud, Y. (2002), Illiquidity and stock returns: cross-section and time-series effects, *Journal of Financial Markets*, 5(1), 31-56.
- Ammann, M., and M. Verhofen (2006), The effect of market regimes on style allocation, *Financial Markets and Portfolio Management,* 20 (3), 309-337.
- Ang, A., and G. Bekaert (2002a), Regime switches in interest rates, *Journal of Business & Economic Statistics,* 20 (2), 163-182.
- Ang, A., and G. Bekaert (2002b), International asset allocation with regime shifts, *Review of Financial studies,* 15 (4), 1137-1187.
- Ang. A., W. Goetzmann and S. Schaefer (2009), Evaluation of Active Management of the Norwegian Government Pension Fund – Global, *Working Paper*.
- Ang A., R.J. Hodrick, Y. Xing and X. Zhang (2006), The cross-section of volatility and expected return, *Journal of Finance*, 61 (1), 259-299.
- Ang A., R.J. Hodrick, Y. Xing and X. Zhang (2009), High idiosyncratic volatility and low returns: International and further US evidence, *Journal of Financial Economics*, 91 (1), 1-23.
- Ang, A., and D. Kristensen (2012), Testing conditional factor models, *Journal of Financial Economics*, 106(1), 132-156.
- Ang, A., and A. Timmermann (2012), Regime Changes and Financial Markets, *Annual Review of Financial Economics,* 4 (1), 313-337.
- Angelidis, T. and N. Tessaromatis (2014a), Global portfolio management under state dependent multiple risk premia, *EDHEC-Risk Institute Working Paper*.
- Angelidis, T., and N. Tessaromatis (2014b), Global Style Portfolios Based on Country Indices, *EDHEC-Risk Institute Working Paper.*
- Aretz, K., S. Bartram, and P. Pope (2010), Macroeconomic risks and characteristic-based factor models, *Journal of Banking & Finance,* 34 (6), 1383-1399.

- Arshanapalli, B. G., E. L. D'Ouville, and W. B. Nelson. (2004), Are Size, Value, and Momentum Related to Recession Risk?, *The Journal of Investing,* 13 (4), 83-87.
- Arshanapalli, B., F.J. Fabozzi, and W. Nelson (2006), The value, size, and momentum spread during distressed economic periods, *Finance Research Letters*, 3(4), 244-252.
- Arshanapalli, B. G., and W. B. Nelson (2007), Small cap and value investing offer both high returns and a hedge, *Journal of Wealth Management*, 9(4), 44-50.
- Asness, C. (1992), Changing Equity Risk Premia and Changing Betas Over the Business Cycle and January, *University of Chicago working paper*.
- Asness, C.S., T.J. Moskowitz, and L.H. Pedersen (2013) Value and momentum everywhere, *Journal of Finance*, 68(3), 929-985.
- Avramov, D., S. Cheng, and A. Hameed (2014), Time-varying momentum payoffs and illiquidity, *Asian Finance Association 2014 Conference Paper*.
- Baker, M., B. Bradley, and J. Wurgler. "Benchmarks as limits to arbitrage: Understanding the low-volatility anomaly." Financial Analysts Journal 67.1 (2011): 40-54.
- Baker, N.L., and R.A. Haugen (2012), Low risk stocks outperform within all observable markets of the world, SSRN Working Paper 2055431.
- Baker, M., and J. Wurgler (2012), Comovement and predictability relationships between bonds and the cross-section of stocks, *Review of Asset Pricing Studies*, 2(1), 57-87.
- Banz, R.W. (1981), The relationship between return and market value of common stocks, *Journal of Financial Economics*, 9(1), 3-18.
- Barberis, N., A. Shleifer, and R. Vishny (1998), A model of investor sentiment, *Journal of Financial Economics*, 49(3), 307-343.
- Barberis, N., and A. Shleifer (2003) Style investing, *Journal of Financial Economics*, 68(2), 161-199.
- Barroso, P. and P. Santa-Clara (2014), Momentum has its moments, *Journal of Financial Economics*, forthcoming.
- Bauer, R., R. Haerden and R. Molenaar (2004), Asset Allocation in Stable and Unstable Times, *Journal of Investing*, 13 (3), 72-80.
- Becker, R.A. (2012), The Variance Drain and Jensen's Inequality, *SSRN Working Paper.*
- Black, A. J., and D. G. McMillan (2005), Value and growth stocks and cyclical asymmetries, *Journal of Asset Management,* 6 (2), 104-116.
- Brown, P., A.W. Kleidon, and T.A. Marsh (1983), New evidence on the nature of size-related anomalies in stock prices, *Journal of Financial Economics*, 12(1), 33-56.
- Bulla, J., S. Mergner, I. Bulla, A. Sesboüé, and C. Chesneau (2011), Markov-switching asset allocation: Do profitable strategies exist?, *Journal of Asset Management*, *12* (5), 310-321.
- Campbell, J.Y., A.W. Lo and A.C. MacKinlay (1997), *The Econometrics of Financial Markets*, Princeton University Press.
- Carhart, M.M. (1997), On Persistence in Mutual Fund Performance, *Journal of Finance*, 52 (1), 57-82.
- Cenesizoglu, T. (2011), Size, book-to-market ratio and macroeconomic news, *Journal of Empirical Finance* 18 (2), 248-270.
- Chan, L.K.C., and J. Lakonishok (2004), Value and growth investing: Review and update, Financial

Analysts Journal, 60(1), 71-86.

- Chang, K-L. (2009), Do macroeconomic variables have regime-dependent effects on stock return dynamics? Evidence from the Markov regime switching model, *Economic Modelling,* 26 (6), 1283-1299.
- Chao, H-Y., C. Collver, and N. Limthanakom (2012), Global style momentum, *Journal of Empirical Finance*, 19(3), 319-333.
- Charoenrook, A., and J.S. Conrad (2006), Identifying risk-based factors, *AFA 2006 Boston Meetings Paper.*
- Chen, H-L., and W. De Bondt (2004), Style momentum within the S&P-500 index, *Journal of Empirical Finance*, 11(4), 483-507.
- Choi, J. (2013), What drives the value premium? The role of asset risk and leverage, *Review of Financial Studies*, 26(11), 2845-2875.
- Chordia, T., and L. Shivakumar (2002) Momentum, business cycle, and time-varying expected returns, *Journal of Finance,* 57 (2), 985-1019.
- Chow, G., E. Jacquier, M. Kritzman, K. Lowry (1999), Optimal portfolios in good times and bad, *Financial Analysts Journal*, 55 (3), 65-73.
- Chui, A.C.W., S. Titman, and K.C.J. Wei (2010), Individualism and momentum around the world, *Journal of Finance*, 65(1), 361-392.
- Cochrane, J. (1999), Portfolio advice of a multifactor world, *Economic Perspectives*, Federal Reserve Bank of Chicago, issue Q III, 59-78.
- Cochrane, J. (2005), *Asset Pricing*, Princeton University Press.
- Cohen R. B., C. Polk, and T. Vuolteenaho (2003), The Value Spread, *Journal of Finance*, 58 (2), 609-642.
- Cooper, I. (2006), Asset pricing implications of nonconvex adjustment costs and irreversibility of investment, *Journal of Finance*, 61(1), 139-170.
- Cooper, M.J., and S. Gubellini (2011), The critical role of conditioning information in determining if value is really riskier than growth, *Journal of Empirical Finance*, 18(2), 289-305.
- Cooper, M.J., R.C. Gutierrez, and A. Hameed (2004), Market states and momentum, *Journal of Finance*, 59(3), 1345-1365.
- Copeland, M. M., and T. E. Copeland (1999), Market timing: Style and size rotation using the VIX, *Financial Analysts Journal*, 55(2), 73-81.
- Coqueret, G., R. Deguest, L. Martellini and V. Milhau (2014), Equity portfolios with improved liability-hedging benefits, *EDHEC-Risk Institute Working Paper*.
- Daniel, K., D. Hirshleifer, and A. Subrahmanyam (1998), Investor psychology and security market under-and overreactions, Journal of Finance, 53(6), 1839-1885.
- Daniel, K. and Moskowitz T. (2013), Momentum Crashes, *Working Paper*.
- Deguest, R., L. Martellini and A. Meucci (2013), Risk Parity and Beyond - From Asset Allocation to Risk Allocation Decisions, *Working Paper*.
- De Long, J. B., A. Shleifer, L.H. Summers, and R.J. Waldmann (1990), Noise trader risk in financial markets. *Journal of Political Economy*, 98(4), 703-738.
- Dimson, E., and P. Marsh. (1999), Murphy's Law and Market Anomalies, *Journal of Portfolio*

Management, 25(2), 53-69.

- Duffie, D. (2001), *Dynamic Asset Pricing Theory*, Princeton University Press.
- Durand, R.B., D. Lim, and J.K. Zumwalt (2011), Fear and the Fama-French Factors, *Financial Management*, 40(2), 409-426.
- Estrella, A., and F.S. Mishkin (1998), Predicting US recessions: financial variables as leading indicators, *Review of Economics and Statistics*, 80(1), 45-61.
- Fama, E., and K. French (1992), The Cross-Section of Expected Stock Returns, *Journal of Finance*, 47 (2), 427-465.
- Fama, E.F., and K.R. French (1993), Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics*, 33(1), 3-56.
- Fama, E.F., and K.R. French (1998), Value versus growth: The international evidence, *Journal of Finance*, 53(6), 1975-1999.
- Fama, E., and K. French (2014), A five factor asset pricing model, *Journal of Financial Economics, Forthcomming.*
- Fong, W.M. (2012), Do expected business conditions explain the value premium?, *Journal of Financial Markets*, 15(2), 181-206.
- Frazzini, A., and L.H. Pedersen (2014), Betting against beta, *Journal of Financial Economics*, 111(1), 1-25.
- Graham, B., and D. Dodd (1934), *Security Analysis*, Mc Graw Hill.
- Gong, Q., M. Liu and Q. Liu (2015), Momentum is really short-term momentum, *Journal of Banking and Finance*, 50, 169-182.
- Goyal, A. and S. Wahal (2015), Is momentum an echo?, *Journal of Financial and Quantitative Analysis,* Forthcoming.
- Griffin, J. M., X. Ji, and J. S. Martin (2003), Momentum investing and business cycle risk: Evidence from pole to pole, *Journal of Finance* 58 (6), 2515-2547.
- Guidolin, M., and A. Timmermann (2008), Size and value anomalies under regime shifts, *Journal of Financial Econometrics,* 6 (1), 1-48.
- Gulen, H., Y. Xing, and L. Zhang (2011) Value versus Growth: Time-Varying Expected Stock Returns, *Financial Management,* 40 (2), 381-407.
- Guo, H., and R. Savickas (2010), Relation between time-series and cross-sectional effects of idiosyncratic variance on stock returns, *Journal of Banking and Finance*, 34(7), 1637-1649.
- Gupta, A., A. Kassam, R. Suryanarayanan, K. Varga (2014), Index Performance in Changing Economic Environments, *MSCI Research Paper*.
- Hahn, J., and H. Lee (2006), Yield spreads as alternative risk factors for size and book-to-market, *Journal of Financial and Quantitative Analysis,* 41(2), 245-269.
- Hamilton, J.D. (1989), A new approach to the economic analysis of nonstationary time series and the business cycle, *Econometrica,* 57, 357-384.
- Haugen, R.A., and N.L. Baker (1991), The efficient market inefficiency of capitalization-weighted stock portfolios, *Journal of Portfolio Management*, 17(3), 35-40.
- Henkel, S.J., J.S. Martin, and F. Nardari (2011), Time-varying short-horizon predictability, *Journal of Financial Economics,* 99 (3), 560-580.

- Hur, J., G. Pettengill, and V. Singh (2014), Market states and the risk-based explanation of the size premium, *Journal of Empirical Finance* 28, 139-150.
- Hwang, S., and A. Rubesam (2013), A behavioral explanation of the value anomaly based on time-varying return reversals, Journal of Banking & Finance, 37(7), 2367-2377.
- Ilmanen, A. (2011), *Expected returns: an investor's guide to harvesting market rewards*. John Wiley & Sons.
- Ilmanen, A., T. Maloney, and A. Ross. (2014), Exploring Macroeconomic Sensitivities: How Investments Respond to Different Economic Environments, *The Journal of Portfolio Management,* 40 (3), 87-99.
- Jegadeesh, N., and S. Titman (1993) Returns to buying winners and selling losers: Implications for stock market efficiency, *Journal of Finance*, 48(1), 65-91.
- Kao, D-L., and R. D. Shumaker (1999), Equity Style Timing, *Financial Analysts Journal*, 55 (1), 37-48.
- Keim, D.B. (1983), Size-related anomalies and stock return seasonality: Further empirical evidence, *Journal of Financial Economics*, 12(1), 13-32.
- Kim, M. K., and D. A. Burnie (2002) The firm size effect and the economic cycle, *Journal of Financial Research,* 25 (1), 111-124.
- Kiyotaki, N., and J. Moore (1997), Credit chains, *Journal of Political Economy*, 105(21), 211-248.
- Kritzman, M., S. Page, and D. Turkington. (2012), Regime shifts: Implications for dynamic strategies, *Financial Analysts Journal,* 68 (3).
- Lakonishok, J., A. Shleifer, and R.W. Vishny (1994), Contrarian investment, extrapolation, and risk, *Journal of Finance*, 49(5), 1541-1578.
- Lamoureux, C.G., and G.C. Sanger (1989), Firm Size and Turn-of-the-Year Effects in the OTC/NASDAQ Market, *The Journal of Finance*, 44(5), 1219-1245.
- La Porta, R. (1996), Expectations and the cross-section of stock returns, *Journal of Finance*, 51(5), 1715-1742.
- La Porta, R., J. Lakonishok, A. Shleifer, and R. Vishny,(1997). Good news for value stocks: Further evidence on market efficiency, *Journal of Finance*, 52(2), 859-874.
- Lettau, M., and J. A. Wachter (2007), Why is long-horizon equity less risky? A duration-based explanation of the value premium, *Journal of Finance*, 62(1), 55-92.
- Lewellen, J. (2002), Momentum and autocorrelation in stock returns, *Review of Financial Studies*, 15(2), 533-564.
- Li, X., C. Brooks, and J. Miffre (2009) The Value Premium and Time-Varying Volatility, *Journal of Business Finance & Accounting*, 36(9), 1252-1272.
- Lo, A.W., and A.C. MacKinlay (1990), When are contrarian profits due to stock market overreaction?, *Review of Financial studies*, 3(2), 175-205.
- Maillard, S., T. Roncalli and J. Teiletche (2010), *The Properties of Equally Weighted Risk Contribution Portfolios*, Journal of Portfolio Management, 36 (4), 60-70.
- Marsh, Ian W. (2000), High-frequency Markov switching models in the foreign exchange market, *Journal of Forecasting,* 19 (2), 123-134.
- Merton, R. (1973), An Intertemporal Capital Asset Pricing Model. *Econometrica,* 41 (5), 867-887.

- Meucci, A., A. Santangelo, and R. Deguest. (2013), Measuring Portfolio Diversification based on Optimized Uncorrelated Factors. *EDHEC-Risk Institute Working Paper*.
- Mitton, T., and K. Vorkink (2007), Equilibrium underdiversification and the preference for skewness, *Review of Financial Studies*, 20(4), 1255-1288.
- Novy-Marx, R. (2012), Is momentum really momentum?, *Journal of Financial Economics*, 103 (3), 429-453.
- Pastor, L., and R.F. Stambaugh (2003), Liquidity Risk and Expected Stock Returns, *Journal of Political Economy*, 111(3), 642-685.
- Perez-Quiros, G., and A. Timmermann (2000), Firm size and cyclical variations in stock returns, *Journal of Finance,* 55 (3), 1229-1262.
- Petkova, R., (2006), Do the Fama–French factors proxy for innovations in predictive variables?, *The Journal of Finance*, 61(2), 581-612.
- Petkova, R., and L. Zhang (2005), Is value riskier than growth?, *Journal of Financial Economics*, 78(1), 187-202.
- Phalippou, L. (2007), Can risk-based theories explain the value premium?, *Review of Finance*, 11(2), 143-166.
- Reinganum, M.R. (1981), Misspecification of capital asset pricing: Empirical anomalies based on earnings' yields and market values, *Journal of Financial Economics*, 9(1), 19-46.
- Roncalli, T. (2013), *Introduction to Risk Parity and Budgeting*. Chapman & Hall, CRC Financial Mathematics Series.
- Rosenberg, B., K. Reid, and R. Lanstein (1985), Persuasive evidence of market inefficiency, *Journal of Portfolio Management*, 11(3), 9-16.
- Ross, S. (1976), The Arbitrage Theory of Capital Pricing, *Journal of Economic Theory,* 13 (3), 341-360.
- Rouwenhorst, K.G. (1998), International momentum strategies, *Journal of Finance*, 53(1), 267-284.
- Schwert, G. W., (2003), Anomalies and market efficiency, *Handbook of the Economics of Finance*, 939-974.
- Sharpe, W. (1964), Capital Asset Prices – A Theory of Market Equilibrium Under Conditions of Risk, *Journal of Finance,* 19 (3), 425-442.
- Sheikh, A.Z., J. Sun (2012), Regime Change: Implications of Macroeconomic Shifts on Asset Class and Portfolio Performance, *Journal of Investing*, 21 (3), 36-54.
- Stivers, C., and L. Sun (2010), Cross-Sectional Return Dispersion and Time Variation in Value and Momentum Premiums, *Journal of Financial and Quantitative Analysis*, 45(4), 987-1014.
- Switzer, L.N. (2012), The behaviour of small cap vs. large cap stocks in recessions and recoveries: Empirical evidence for the United States and Canada, *North American Journal of Economics and Finance*, 21(3), 332-346.
- Teo, M., and S-J. Woo (2004), Style effects in the cross-section of stock returns, *Journal of Financial Economics*, 74(2), 367-398.
- Tu, J. (2010) Is regime switching in stock returns important in portfolio decisions?, *Management Science,* 56 (7), 1198-1215.
- Van Dijk, M. A. (2011), Is size dead? A review of the size effect in equity returns, *Journal of*

Banking & Finance, 35(12), 3263-3274.

- Wang, K.Q., and J. Xu (2015), Market volatility and momentum, *Journal of Empirical Finance*, 30, 79-91.
- Welch, I., and A. Goyal (2008), A comprehensive look at the empirical performance of equity premium prediction, *Review of Financial Studies,* 21 (4), 1455-1508.
- Zhang, L. (2005), The value premium, *Journal of Finance*, 60(1), 67-103.
- Zhang, Q. J., J. Hopkins, S. Satchell and S. Schwob (2009), The link between macro-economic factors and style returns, *Journal of Asset Management,* 10 (5), 338-355.