Equity Portfolios with Improved Liability-Hedging Benefits

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Abstract

This paper analyses the question of the *feasibility* and *desirability* for a liability-driven investor to hold an equity portfolio engineered to exhibit enhanced liability-hedging properties versus holding a broad equity index. We first show within a continuous-time dynamic portfolio selection model that investor welfare is not only increasing in the Sharpe ratio of the performance portfolio and in the correlation of the liability-hedging portfolio with the liabilities, as suggested by the *fund separation theorem*, but it is also increasing in the correlation between the performance portfolio and the liabilities. The practical implication of this *fund interaction theorem* is that liability-driven investors will in general benefit from improving hedging characteristics of their performance portfolio, unless this improvement is associated with an exceedingly large opportunity cost in terms of performance. In a second part of the paper, we report empirical evidence of the presence of strong cross-sectional dispersion in liability-hedging characteristics of individual stocks within the S&P500 universe. We also demonstrate that liability-driven investors may derive substantial welfare benefits from the joint selection of low volatility and high dividend yield stocks, a procedure that is found to lead to economically and statistically significant improvements in liability hedging benefits compared to the use of a broad equity market index. These benefits are further enhanced when the selected stocks are combined with variance minimizing weights, a weighting mechanism that contributes to a marginal improvement in hedging benefits and a substantial increase in risk-adjusted performance. Our findings are robust with respect to changes in the sample period, in the number of stocks used in the selection procedure, in the duration of the liabilities, and in the presence of inflation-indexation in the liability streams.

1 Introduction

The *fund separation theorem*, which is a fundamental cornerstone of dynamic asset pricing theory, suggests that risk and performance are two conflicting objectives that are best managed when managed separately within two dedicated building blocks. In an asset-only context, it implies that all investors should allocate some fraction of their wealth to the risk-free asset and the remainder to the optimal (that is maximum Sharpe ratio) risky portfolio (Tobin (1958)). In an asset-liability management (ALM) context (e.g. a pension plan facing liability commitments), the fund separation theorem translates into *liability-driven investing* (LDI in short), a disciplined investment framework that advocates splitting an investor's wealth between a dedicated liability-hedging portfolio (LHP) and a common performance-seeking portfolio (PSP) (see Martellini (2006) or Martellini and Milhau (2012)).[1] While the LDI paradigm implies that investor welfare should depend on how successful each building block is at delivering what it has been designed for, namely performance benefits for the PSP and hedging benefits for the LHP, the intuition suggests that the *interaction* between performance and hedging motives should also play an important role. For example, it is expected that an investor who is given a choice between two PSPs with approximately identical risk-adjusted performance characteristics but extremely different liability-hedging properties should opt for the one with more attractive liability hedging benefits.

A formal analysis of this effect within the context of a continuous-time dynamic asset allocation model is provided in the theoretical section of the present paper (Section 2), where we investigate whether investor welfare can be improved by the design of PSPs with improved liability-hedging properties. This question may at first glance seem inconsistent with the key insight gained from the fund separation theorem, which advocates a separate, as opposed to a joint, focus on performance and hedging in the design of the optimal investment strategy. By introducing a formal decomposition of liability-driven investors' welfare, we are however able to show that it is not only increasing in the Sharpe ratio of the PSP and in the correlation of the LHP with the liabilities, as suggested by the fund separation theorem, but it is also increasing in the correlation between the PSP and the liabilities (see Proposition 2.2). One key practical implication of this *fund interaction theorem* is that investors such as pension funds will in general benefit from improving hedging characteristics of their PSP, unless this improvement is associated with an exceedingly large opportunity cost in terms of risk-adjusted performance. Ultimately, the net impact will be positive or negative depending on the relative strength of the following two competing effects. On the one hand, the equity benchmark with improved hedging benefits can represent a higher fraction of the investor's portfolio for a given ALM risk budget; on the other hand, the equity benchmark with improved hedging properties may have a lower risk-adjusted performance. Hence the trade-off is between an increase in performance due to a higher allocation to risky assets, and a decrease in risk-adjusted performance due to a lower reward

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[1]In an ALM context, cash is also used as an additional building block, which allows, for example, the use of leveraged LDI strategies.

for each dollar invested.

It is useful at this stage to emphasise that the fund interaction theorem and the fund separation theorem are not mutually inconsistent; in fact they co-exist within the framework of liability-driven investing, which the aforementioned results do not contradict, even though they do advocate a focus on the interaction between performance and hedging motives. In particular, one should indeed acknowledge that it follows from the uniqueness of the maximum Sharpe ratio (MSR) portfolio given a set of parameter values, that investor long-term welfare is (by definition of the LDI strategy being the optimal strategy in the presence of liability constraints) the highest when the Sharpe ratio of the PSP is maximised (without taking into account the interaction with the liabilities) and when the (squared) correlation of the LHP with the liabilities is maximised (without taking into account its risk-adjusted performance). In this context, it may seem unclear how an improvement in investor welfare can be generated by the use of a PSP with improved hedging properties, if this improvement should come at the cost of a lower Sharpe ratio. In practice, however, uncertainty about parameter values, and in particular expected returns (see Merton (1980)), implies that one cannot form ex-ante a perfect MSR PSP from individual securities or asset classes. As a result, investors are at best left with reasonably good proxies for efficient PSPs, with hopefully decent risk-adjusted performance benefits. Hence, while the true MSR PSP generates the highest level of welfare for all investors (fund separation theorem), the relative merits of various competing heuristic proxies for performance portfolios need to be empirically assessed as a function not only of their performance properties but also of their hedging properties, by virtue of the fund interaction theorem. Another key limitation in terms of real-world implementation is the presence of leverage constraints, which implies that most underfunded pension funds cannot use as much leverage as would be required to fully hedge their liabilities. In practice, pension funds end up investing all their assets in a zero- or low-leverage portfolio mostly containing stocks and bonds, with a key trade-off between a dominant allocation to equities (say a 60/40 stock/bond split), which generates attractive levels of expected returns, but also implies high levels of funding ratio volatility, or a more moderate equity allocation (say a 40/60 stock/bond split) which requires lower ALM risk budgets but correspondingly also generates lower upside potential. In this context, the question arises whether it would it make sense for a pension fund to hold a customised equity portfolio engineered to exhibit enhanced liability-hedging properties versus holding an off-the-shelf broad equity index. Intuition indeed suggests that a better alignment of the PSP with respect to the liabilities would lead to an increased allocation to stocks for the same level of volatility of the funding ratio, which in turn would generate higher access to the equity risk premium.

In addition to the presence of parameter uncertainty and leverage constraints also remain a number of additional degrees of freedom, which explain that the question of the joint assessment of the performance and hedging benefits of a proxy for the PSP is of practical relevance. The first of these degrees of freedom is related to the menu of asset classes allowed to enter the composition of the PSP and LHP portfolios. In fact, by enlarging the investment universe, investors may be able to simultaneously improve the performance and hedging benefits of the PSP, which would result in a higher

welfare compared to a situation with a more limited menu of available asset classes. For example, the introduction in the asset mix of inflation-linked bonds may lead not only to improvements to the LHPs for investors facing inflation-linked liabilities. It may also lead to improvements to the PSP since the introduction of these bonds will, in general, result in an increase in risk-adjusted performance (due to low correlations of inflation-linked bonds with other asset classes such as equities) as well as an increase in liability-friendliness (see Section 3 for a formal definition of this concept). Another important degree of freedom is the choice of the benchmark used to represent the performance of a given asset class. In this context, the question arises of the benefits/costs associated with switching to one benchmark to another benchmark with lower or higher liability-hedging and performance-seeking benefits. In particular, while the traditional choice is to use a cap-weighted (CW) index as a benchmark for equities, there is substantial empirical evidence that non cap-weighted benchmarks, sometimes also known as smart beta benchmarks, may enjoy a higher level of diversification, and as such tend to have higher Sharpe ratios and expected returns compared to corresponding CW benchmarks (see for example Amenc et al. (2012)). Some of these benchmarks may also have improved liability-hedging benefits (see for example a formal analysis of the interest rate hedging properties of global minimum variance benchmarks by Fishwick (2013)), so that a move away from the use of CW benchmarks may result in an improvement on both performance and hedging dimensions, with no contradiction with respect to the fund separation theorem. One of the goals of the empirical section of this paper (Section 4) is precisely to provide a quantitative measure of welfare improvements expected from switching from a standard off-the-shelf CW equity benchmark to a dedicated equity benchmark designed to exhibit above-average liability-hedging properties. There is finally another key feature to take into account in practice, which is the presence of a number of regulatory or implementation constraints. For instance, investors may be constrained to only hold fixed-income instruments in the LHP, for example because of a regulatory or other concern over short-term risk relative to liabilities driven by a mismatch in interest rate risk exposure. This constraint a priori implies a loss of long-term efficiency compared to an otherwise unconstrained optimal long-term LHP, which may include for example risky assets with attractive inflation-hedging exposure for inflation-linked liabilities. In this context, investors may benefit from having a PSP which holds more of these risky assets with attractive long-term (inflation) hedging benefits compared to what would be otherwise needed if they were only maximising the Sharpe ratio.

Overall, we argue that the fund interaction theorem is not inconsistent with the fund separation theorem, which it complements by emphasising the benefits of having, if and when possible, an alignment, as opposed to a conflict, between the performance and hedging motives. In the empirical section of this paper (Section 4), we analyse the practical implications of this fund interaction theorem in terms of the design of liability-friendly equity benchmarks (i.e. equity benchmarks with improved liability-hedging benefits). In this context, we show that very substantial increases in investor welfare would come from switching from a standard off-the-shelf CW equity benchmark to an equity benchmark designed to exhibit above-average liability hedging properties. We focus on equities since this

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asset class is arguably the dominant asset class within the PSP for most institutional investors. We cast the analysis at the individual stock level, as opposed to sticking to the sector level, given the presence of a very substantial levels of cross-sectional dispersion in interest rate and inflation hedging benefits across individual stocks (see literature review in Section 3). More precisely, we consider two alternative approaches to the definition of liability-friendliness. Our first approach is of a statistical nature and relies on the capacity of a stock to track a given liability proxy, which we model as a constant maturity nominal or inflation-linked bond index. In this setting with a focus on risk factor matching, a stock will be said to be liability-friendly if the tracking error of the stock returns with respect to the returns on the liability proxy is low. Given the decomposition of the tracking error in two components, one that is related to the portfolio volatility and one that is related to the portfolio correlation with the liability proxy, a low tracking error can be achieved either if the volatility of the stock is low and/or if the correlation between the stock and the liability proxy is high. Therefore, liability-friendly stocks are expected to fulfil either one or two of these requirements. The second approach which we propose for the characterisation of liability-friendly stocks hinges on their ability to replicate fixed pension liability cash-flows. This cash-flow matching focus can be best achieved by stocks and portfolios of stocks that enjoy a high and stable dividend yield.

In an empirical analysis, we use these insights to address a number of practically important questions related to the impact on investor welfare of potential improvements of liability-hedging properties of equity portfolios held by pension funds. Our key quantitative findings can be summarised as follows. In the cross-section, the main driver of liability-friendliness in a statistical factor-matching sense is volatility. Over our 1975-2012 sample period for the S&P 500 universe, the equally-weighted (EW) portfolio of the 20% of stocks with the lowest volatilities has a tracking error of 14.6% with respect to our liability proxy while the EW portfolio of the 20% of stocks with the highest volatilities is almost twice as large at 27.8%. This spectacular improvement in tracking error does not only emanate from a lower portfolio volatility; it is also linked so a strong increase in correlation with the liabilities. Hence, the selection of low volatility stocks generates a positive 7.7% correlation with the liability proxy, while a selection of high volatility stocks would generate a negative correlation or -6.7%. Intuitively, this improvement can be traced down to the fact that low volatility stocks, which tend to be low dividend uncertainty stocks, are the stocks that tend to be the closest approximations to fixed-income securities, and as a result, the best approximation to bond-like liabilities. These results are robust with respect to changes in the number of selected stocks, changes in the sample period, changes in the duration of the liability proxy, or the introduction of inflation-indexation. Addressing the focus on liability-hedging through a double-sort procedure, starting with the 200 highest dividend yield stocks, selecting the 100 lowest volatility stocks amongst them, and subsequently performing a minimum variance optimisation, leads to further improvements in the liability-friendliness of the selected portfolios.

As a result of the resulting improvement in liability-hedging benefits, a liability-driven investor allocating for example 40% to equities on the basis of a CW equity benchmark can allocate as much as 53.3% to a minimum variance portfolio of selected stocks from the aforementioned double-sort

procedure for the same volatility of the funding ratio, and for a maximum drawdown relative to the liabilities going down from 33.4% to 25.7%. The resulting increase in equity allocation for the same ALM risk budget, combined with an improved risk-adjusted performance of the dedicated equity benchmark with respect to the S&P 500 index, leads to an improvement in performance reaching close to 160 basis points annualised over the 1975-2012 sample period. For inflation-linked liabilities, we find (over a shorter period starting in 1999) that the use of an equity benchmark with improved liability-hedging benefits generates an annualised excess return of 270 basis points for the same funding ratio volatility compared to the use of a standard CW S&P 500 benchmark.

The rest of the paper is organised as follows. Section 2 introduces a formal framework suited for an empirical investigation of whether investor welfare can be enhanced by the design of performance portfolios with improved liability-hedging properties. In Section 3, we introduce a number of formal measures of liability-friendliness. In Section 4, we report empirical evidence of high levels of cross-sectional dispersion in these measures, discuss the implications in terms of the selection of stocks exhibiting above average liability-friendliness, and also analyse the impact on investor welfare of the use of an equity benchmark with improved liability-hedging benefits. Section 5 presents some conclusions and suggestions for further research.

2 Fund Separation Theorem versus Fund Interaction Theorem

In this section, we describe a formal continuous-time model in which investor welfare is measured by expected utility and can be computed analytically. We introduce two measures of welfare gain, which can be used to assess the benefits of increasing the correlation between the performance portfolio and liabilities within a liability-driven investing strategy.

2.1 The Investment Model

Uncertainty in the economy is represented by a filtered probability space (X,F,P), where F is a sigma-algebra on X, and P is a probability measure that represents investor's beliefs. The finite time span is denoted with $[0,T]$, where T can be thought of the investment horizon. The probability space supports a d-dimensional Brownian motion $\underline{z}$, d being the number of independent sources of risk in the economy. The filtration generated by this Brownian motion is denoted with $(F_t)_{0 \le t \le T}$: the sigma-algebra F_t represents the information available to the investor on date t. The nominal short-term interest rate on date t (with infinitesimal borrowing or lending horizon) is denoted with r_t.

The investment universe consists of N "locally risky" assets, whose prices are denoted with $S_{1t}, \ldots, S_{Nt}$. They follow the dynamics:

$$\frac{dS_{it}}{S_{it}} = [r_t + \sigma_{it}\lambda_{it}]dt + \underline{\sigma}'_{it}d\underline{z}_t,$$

where λ_{it} is the Sharpe ratio, $\underline{\sigma}_{it}$ is the $d \times 1$ volatility vector, and σ_{it} is the scalar volatility.[2] There also

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[2]We use underbars to denote vectors and matrices.

exists a "locally risk-free" asset whose price is the value of the continuously compounded short-term rate:

$$S_{0t} = \exp\left[\int_0^t r_s ds\right].$$

For tractability purposes, we make the following assumptions:

- The short-term interest rate follows the Vasicek model (Vasicek (1977)) :

$$dr_t = a(b - r_t)dt + \underline{\sigma}_r' d\underline{z}_t;$$

- The Sharpe ratios and the correlations of assets with each other and with the short-term interest rate are constant.

We let λ_r denote the (constant) price of interest rate risk. For further use, we also introduce the volatility matrix of the risky assets:

$$\underline{\sigma}_t = (\underline{\sigma}_{1t} \ldots \underline{\sigma}_{Nt}).$$

It is useful to factor out the scalar volatilities (which are possibly stochastic). To this end, consider the correlation matrix of the risky assets, $\underline{\Omega}$, and its Cholesky decomposition, $\underline{\Omega} = \underline{UU}'$. $\underline{U}$ is thus the matrix of the normalised volatility vectors, $\underline{\rho}_i = \frac{1}{\sigma_{it}}\underline{\sigma}_{it}$, which are constant by assumption. Letting $\underline{V}_t$ be the diagonal matrix of scalar volatilities, we thus have that:

$$\underline{\sigma}_t = \underline{UV}_{t.}$$

The instantaneous covariance matrix is thus $\underline{\Sigma}_t = \underline{\sigma}_t'\underline{\sigma}_t$, and the vector of expected excess returns on the risky assets can be written as $\underline{\mu}_t = \underline{V}_t\underline{\Lambda}$, where $\underline{\Lambda}$ is the vector that contains all Sharpe ratios. Following He and Pearson (1991), a price of risk vector is defined as any vector process $(\underline{\lambda}_t)_{0 \leq t \leq T}$ that satisfies, for all t:

$$\underline{\sigma}_t'\underline{\lambda}_t = \underline{\mu}_t.$$

Of particular interest is the "spanned price of risk vector", which is the only price of risk vector that falls in the span of the volatility matrix. It is defined as:

$$\underline{\lambda} = \underline{\sigma}_t(\underline{\sigma}_t'\underline{\sigma}_t)^{-1}\underline{\mu}_t = \underline{U}\underline{\Omega}^{-1}\underline{\Lambda}.$$

The second expression shows that this vector is actually constant: uncertainty in volatilities is cancelled out from the product, and the spanned price of risk vector is a function of correlations and Sharpe ratios, which are constant. It is important to note that it is not the only price of risk vector, unless the market is dynamically complete: this condition is equivalent to having $d = N$ (see He and Pearson (1991) and Duffie (2001)), but if $d > N$, that is, if the number of sources of risk in the economy exceeds the number of traded risky assets, then the market is incomplete.

A portfolio strategy is described by a weight vector process $(\underline{w}_t)_{0 \leq t \leq T}$, where $\underline{w}_t$ is the $N \times 1$ vector of weights allocated to the risky assets at date t. The sum of the elements of this vector is not necessarily equal to 1, in which case cash can be used to make up the balance: if the sum of weights invested in risky assets is less than 1, one takes a long position in cash; otherwise one takes a short position (borrowing). Let us denote the wealth process with $(A_t)_{0 \leq t \leq T}$, A_0 being the capital invested at time 0. Using the previously introduced notations, the intertemporal budget constraint can be written as:

$$\frac{dA_t}{A_t} = [r_t + \underline{w}_t' \underline{\sigma}_t' \underline{\lambda}]dt + \underline{w}_t' \underline{\sigma}_t' d\underline{z}_t.$$

Investor's preferences for high return and low variance can be captured within the expected utility framework. We take the utility function to be the Constant Relative Risk Aversion (CRRA) one, which is a popular choice in the asset allocation literature:

$$U(x) = \frac{x^{1-\gamma}}{1 - \gamma},$$

where γ is the risk aversion coefficient, which is assumed to be greater than or equal to 1. The case of a unit γ corresponds to the logarithmic utility function: the investor is then only concerned with the expected return of its portfolio, not with the variance.

In order to model the situation of a defined-benefit pension fund, we assume that the investor has liabilities, whose value is represented by the value of a constant maturity nominal bond: this process is thus the value of a continuous roll-over of nominal bonds, which all have the same maturity τ. In the Vasicek model, the dynamics of the constant-maturity bond reads:

$$\frac{dL_t}{L_t} = [r_t + \sigma_L \lambda_L]dt + \underline{\sigma}_L' d\underline{z}_t,$$

where the Sharpe ratio is $\lambda_L = -\lambda_r$, and $\underline{\sigma}_L$ is the volatility vector of liabilities, given by:

$$\underline{\sigma}_L = -\frac{1 - e^{-a\tau}}{a} \underline{\sigma}_r = -D(\tau)\underline{\sigma}_r.$$

This expression for the volatility vector shows that the correlation between the liability process and any risky asset is the negative of the correlation between the interest rate and the risky asset: this correlation is thus constant. In what follows, we let $\underline{R}_L$ denote the $N \times 1$ vector of correlations between the risky assets and the liability process.

2.2 Optimal Portfolio Rule

Because the investor is concerned with the performance of the portfolio relative to its liabilities (not the absolute performance), utility is not derived from wealth itself, but from the funding ratio. Thus, the investor's objective is to find the portfolio strategy that gives the highest expected utility from

terminal funding ratio:

$$\underline{w}^* = \underset{\underline{w}}{argmax}\, E\left[U\left(\frac{A_T}{L_T}\right)\right].$$

The optimal policy is given by the following proposition, the proof of which can be found in Martellini and Milhau (2012).

Proposition 2.1. (**Optimal Strategy**). *The utility-maximising strategy is:*

$$\underline{w}_t = \frac{\lambda_{MVP}}{\gamma \sigma_{MVP,t}} \underline{w}_{PSP,t} + (1 - \frac{1}{\gamma})\beta_{L/LHP,t}\underline{w}_{LHP,t,}$$

where

- *$\underline{w}_{MVP,t}$ is the mean-variance efficient performance-seeking portfolio (PSP), that maximises the instantaneous Sharpe ratio:*
$$\underline{w}_{PSP,t} = \frac{1}{\underline{1}'\Sigma_t^{-1}\underline{\mu}_t}\Sigma_t^{-1}\underline{\mu}_t,$$

- *λ_{PSP} and $\sigma_{PSP,t}$ are the Sharpe ratio and the volatility of the PSP:*

$$\lambda_{PSP} = \sqrt{\underline{\Lambda}'\underline{\Omega}^{-1}\underline{\Lambda}},$$

$$\sigma_{PSP,t} = \frac{\lambda_{MVP}}{\underline{1}'\Sigma_t^{-1}\underline{\mu}_t},$$

- *$\underline{w}_{LHP,t}$ is the liability-hedging portfolio (LHP), that maximises the instantaneous correlation with liabilities:*
$$\underline{w}_{LHP,t} = \frac{1}{\underline{1}'\Sigma_t^{-1}\underline{\sigma}_t'\underline{\sigma}_L}\Sigma_t^{-1}\underline{\sigma}_t'\underline{\sigma}_L,$$

- *$\beta_{L/LHP,t}$ is the beta of liabilities with respect to the LHP:*

$$\beta_{L/LHP,t} = \underline{1}'\Sigma_t^{-1}\underline{\sigma}_t'\underline{\sigma}_L.$$

The optimal strategy is of the liability-driven investing type, since it is a linear combination of a PSP, which is optimally taken to be the MSR portfolio, and a portfolio whose objective is to replicate liability value as closely as possible (the LHP). The properties of the optimal policy have been extensively discussed in the literature (see e.g. Detemple and Rindisbacher (2010)). For instance, the allocation to the mean-variance efficient PSP is increasing in the Sharpe ratio of this portfolio, and the allocation to the LHP is increasing in the beta coefficient, hence in the correlation between the LHP and liabilities. In other words, the allocation to each of the two building blocks is an increasing function of the criterion that this block is designed to maximise.

2.3 Fund Interaction Theorem

The indirect utility is defined as the maximum expected utility that can be achieved given the available capital (A_0) and current market conditions (here, summarised by the risk-free interest rate r_0).[3] However, as the level of utility in itself is hard to interpret, we introduce a closely related concept, which is the Logarithmic Utility Gain (in short, LUG). This quantity is defined as the logarithm of the factor by which the initial capital must be multiplied for an investment in the risk-free asset to produce the same expected utility as the optimal strategy: in other words, the investor is indifferent between investing A_0 in cash and the risky assets, following the optimal rule, and investing $A_0 \times e^{LUG}$ in cash. From this definition, it is easy to see that the "annualised LUG", defined as $\frac{LUG}{T}$, has the interpretation of an excess return that must be added to the risk-free rate for an investment in cash to lead to the same expected utility as the optimal strategy. Note that the LUG is independent from the current funding ratio, a property that follows from the choice of the CRRA utility function.

By construction, the LUG is always positive, unless an investment in cash only turns out to be the optimal policy, in which case it is zero. The LUG is given by the following proposition, which we interpret as a *fund interaction theorem*. The proof of the proposition can be found in Appendix A.

Proposition 2.2. (**Fund interaction theorem 1**). *The LUG associated with the optimal strategy is:*

$$LUG^* = \frac{1}{2\gamma}\lambda_{PSP}^2 T + \frac{(1-\gamma)^2}{2\gamma}\sigma_L^2\rho_{LHP,L}^2 T + (1 - \frac{1}{\gamma})\sigma_L\lambda_{PSP}\rho_{PSP,L}T,$$

where:

- $\rho_{MVP,L}$ *is the correlation between the PSP and liabilities:*

$$\rho_{PSP,L} = \frac{\Lambda'\Omega^{-1}R_L}{\sqrt{\underline{\Lambda}'\,\Omega^{-1}\underline{\Lambda},}},$$

- $\rho_{LHP,L}$ *is the correlation between the LHP and liabilities:*

$$\rho_{LHP,L} = \sqrt{\underline{R_L'}\Omega^{-1}\underline{R_L}}.$$

This proposition establishes a decomposition of investor welfare across the two risky funds of the fund separation theorem. The first term is a pure contribution from the mean-variance efficient portfolio, which is proportional to the squared Sharpe ratio of this fund, regardless of its hedging properties. The second term is a pure contribution from the LHP; it is proportional to the squared

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[3]Nielsen and Vassalou (2006) show that the opportunity set (a synonym for market conditions) is characterised by the state variables that impact the intercept and the slope of the intertemporal capital market line (ICML): the intercept is the risk-free rate, and the slope is the Sharpe ratio of the PSP. Because the Sharpe ratios and the pairwise correlations of assets are constant, the slope is constant, and the only source of uncertainty that affects the position of the ICML is the current risk-free rate.

correlation between the portfolio and the liabilities that it intends to hedge. The last term involves the correlation of the PSP with the liabilities, and is thus related to how attractive (or unattractive) the hedging properties of the PSP may be. The interpretation of this cross-term is straightforward: investor welfare is enhanced in case the PSP has attractive hedging properties. Everything else equal, the LUG is increasing in the product $\lambda_{PSP}\rho_{PSP,L}$. As long as the Sharpe ratio of the PSP is positive, this implies that investor welfare is increasing in the correlation between this portfolio and the liabilities. In sum, the cross-term describes the interactions between the various funds/motives, and how these interactions contribute to investor welfare.

In fact, Proposition 2.3 below shows that the interaction term can be equally interpreted as being related to the hedging benefits of the performance portfolio or to the performance benefits of the LHP. The proof is straightforward given the identity

$$\lambda_{LHP} = \frac{\Lambda'\Omega^{-1}R_L}{\sqrt{\underline{R}_L'\Omega^{-1}\underline{R}_L}}.$$

Proposition 2.3. (**Fund interaction theorem 2**). *With the same notation as above, the following relation holds:*

$$\lambda_{PSP}\rho_{PSP,L} = \lambda_{LHP}\rho_{LHP,L} = \underline{\Lambda}'\underline{\Omega}^{-1}\underline{R}_L.$$

In other words, while each building block is by construction the best at what it has been specifically designed for: (risk-adjusted) performance (maximising Sharpe ratio) or liability-hedging, the two portfolios are strictly equivalent in terms of the product of their relevant characteristics; the relative domination of the performance portfolio in terms of the Sharpe ratio is exactly compensated by the relative domination of the LHP in terms of the correlation with liabilities.

2.4 A Formal Welfare Gain Measure

As explained in the introduction, uncertainty about parameter values (and in particular expected returns) implies that one can never hold the true MSR portfolio. For example, it is standard practice for investors to use a CW index as an equity benchmark for the equity allocation within the PSP. While the true MSR PSP generates the highest level of welfare for all investors (fund separation theorem), various competing heuristic proxies for performance portfolios need to be empirically assessed as a function of their performance but also hedging properties (fund interaction theorem). In particular, one might be interested in analysing whether investor welfare will be increased or decreased if moving away from a CW equity benchmark to a dedicated equity benchmark constructed to have superior liability-hedging benefits.

Formally, LUG as a quantitative measure of investor welfare can be used to compare two different investment universes. To fix the ideas, and to introduce a framework consistent with the empirical part of this paper, let us consider two investment universes, referred to as 0 and 1.

- Universe 0 consists of cash, a standard CW equity index (e.g. the S&P 500), plus a bond index that is assumed to perfectly replicate liabilities (i.e. has a correlation of 1 with liabilities). The LHP is thus fully invested in this bond index. For notational clarity, we denote by PSP_0 the PSP constructed from the equity and bond indices and by LUG_0^* the LUG associated with the optimal strategy in this universe.

- Universe 1 consists of cash, an alternative equity benchmark, constructed from a sub-universe of the S&P 500 universe, with a possibly different weighting scheme (see Section 3 for more details about the stock selection procedure and Section 4 for more details on the alternative weighting schemes), and again the perfect liability-matching bond portfolio. We denote the corresponding PSP by PSP_1 and the LUG by LUG_1^*.

If the difference $\Delta LUG^* = LUG_1^* - LUG_0^*$ is positive, then Universe 1 is "better" than Universe 0 in the sense that the maximum expected utility achieved in this universe is higher. A negative difference means that the selection operation has led to a lower indirect utility, hence that Universe 0 is preferable. The size of the difference can be interpreted in terms of variation of initial contribution: by definition of the LUG, the investor is indifferent between investing A_0 optimally in Universe 1, and investing $A_0 \times e^{\Delta LUG^*}$ optimally in Universe 0.

In fact, it can be shown that if the following (heroic) conditions are simultaneously satisfied, then the difference in welfare related to different equity benchmarks will only depend (positively) on the Sharpe ratio of the competing equity benchmark, or more precisely, on the Sharpe ratio that will be achieved for the PSP constructed with these equity benchmarks, and not on the correlation of these benchmarks with the liabilities:

A1: A perfect liability hedging portfolio exists; in other words, the maximum correlation with the liabilities achieved with the given menu of asset classes is strictly less than 1.

A2: The investor holds the true MSR portfolio based on the available assets as a PSP.

A3: The investor holds the optimal combination of the true MSR portfolio and the perfect LHP.

The proof of this result follows as a corollary of Proposition 2.3, which states that the product of the Sharpe ratio and correlation with liabilities is the same for the PSP and the LHP, a result which holds only for optimally designed building blocks, that is, for the MSR portfolio and the maximum correlation portfolio. In both universes, the LHP is fully invested in the bond index, so that the product $\lambda_{LHP}\rho_{LHP,L}$ is constant. As a consequence, the product $\lambda_{PSP}\rho_{PSP,L}$ is the same for both universes, which leads to:

$$\Delta LUG^* = \frac{1}{2\gamma}(\lambda_{PSP_1}^2 - \lambda_{PSP_0}^2)T,$$

Thus, ΔLUG^* is only a function of the Sharpe ratios of the PSPs, and does not depend on their respective correlations with liabilities, as it should be in accordance with the fund separation theorem.

Conversely, if any one of these conditions is violated, then changes in investor welfare when shifting from the standard S&P 500 equity benchmark to an alternative equity benchmark, the change in

investor welfare correlation will not only depend upon the Sharpe ratios of the standard and alternative equity benchmarks, but also on their correlation with the liabilities. Amongst the many reasons why the assumptions above are expected to be violated in practice, the following stand out. First, regarding assumption A1, investors can have access to a perfect LHP only if no unhedgeable sources of risk exist. In practice, the presence of longevity risk or inflation risk for inflation-linked liabilities, together with capacity constraints in longevity derivatives or inflation-linked instruments capacity constraints imply that LHPs will only allow for an imperfect match of liability risk. Even when interest rate risk is the only factor impacting changes in pension liabilities, it happens that existing instruments (nominal bonds, futures contracts or swap contracts) do not allow for a perfect liability hedge because they often have a duration lower than that of the liabilities. Turning to assumption A2, the presence of parameter uncertainty, particularly severe for expected return parameters (Merton (1980)) implies that it is impossible in practice to hold the perfect MSR portfolios, and heuristic proxies are used instead for that portfolio. Finally, assumption A3 is most often violated in practice, not only for the presence of parameter uncertainty which again makes it impossible for investors to measure the volatility and Sharpe ratio of their PSP, but also the presence of leverage constraints very often prevent investors from holding a theoretically derived optimal allocation to the PSP and LHP, which typically involves for reasonable parameter values an excessive amount of leverage.

In this context, the relevant question in general becomes the following in practice: For a liability-driven investor who expects to allocate a given fraction $x\%$ of the assets to equities, should there be welfare gains to be expected from moving away from a standard CW benchmark to an alternative equity benchmark? Answering this question depends not only on the differences in risk-adjusted performance, but also on differences in correlation with the liabilities between the two benchmarks.

More precisely, in anticipation of the empirical exercises from Section 4, we seek to compare the improvement (or deterioration) in investor welfare that follows from the switch to a different equity benchmark in the context of heuristic fixed-mix strategies. To this end, let us consider a liability-driven investing (LDI) strategy invested in an equity benchmark S, and a bond benchmark B, with respective weights x and $1 - x$ (x is a parameter lying between 0 and 1, thus excluding the use of cash). We maintain the assumption that the bond benchmark can be taken as a perfect match for the liability portfolio. In this simple setting with only two asset classes, one of which is a perfect proxy for the liabilities, the family of LDI strategies can be obtained by all possible combinations of the two building blocks, with respective weights x and $1 - x$ (x is a parameter lying between 0 and 1 since we rule out leverage and short-selling in this exercise):

$$\underline{w} = x\underline{w}_S + (1 - x)\underline{w}_B.$$

In order to compute the expected utility associated with this strategy, we need to make the additional assumption that the volatilities of the two risky assets, S and B, are constant.[4]

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[4]The reason for making this assumption is that we need the volatility vector of the wealth process, $\underline{\sigma}_t\underline{w}$, to be a

As for the optimal strategy, it is possible to define the LUG as the logarithm of the factor by which the initial wealth must be multiplied for the investor to be indifferent between investing A_0 in the LDI strategy and $A_0 \times e^{LUG}$ in cash only. The following proposition provides a closed-form expression for the LUG, which is no longer necessarily positive. The proof of the proposition is located in Appendix B.

Proposition 2.4. (**LUG of Fixed-Mix Strategy**). *Let A_T be the terminal wealth generated by investing A_0 in the fixed-mix LDI strategy and A_T^c be that generated by investing A_0 in cash. The LUG associated with the fixed-mix LDI strategy is:*

$$LUG^{fm} = E\left[\ln\frac{A_T}{L_T}\right] + \frac{1-\gamma}{2}V\left[\ln\frac{A_T}{L_T}\right] - \left\{E\left[\ln\frac{A_T^c}{L_T}\right] + \frac{1-\gamma}{2}V\left[\ln\frac{A_T^c}{L_T}\right]\right\},$$

where:

- *The variances of the log funding ratios are:*

$$V\left[\ln\frac{A_T}{L_T}\right] = [x^2\sigma_S^2 + (1-x)^2\sigma_B^2 + 2x(1-x)\sigma_S\sigma_B\rho_{S,B} - 2x\sigma_S\sigma_L\rho_{B,L} - 2(1-x)\sigma_B\sigma_L\rho_{B,L} + \sigma_L^2]T;$$

$$V\left[\ln\frac{A_T^c}{L_T}\right] = \sigma_L^2 T;$$

- *The expected log funding ratios are:*

$$E\left[\ln\frac{A_T}{L_T}\right] = \ln\frac{A_0}{L_0} + \left[x\mu_S + (1-x)\mu_B - \sigma_L\lambda_L + \frac{\sigma_L^2}{2}\right]T$$
$$- \frac{1}{2}\left[x^2\sigma_S^2 + (1-x)^2\sigma_B^2 + 2x(1-x)\sigma_S\sigma_B\rho_{S,B}\right]T;$$

$$E\left[\ln\frac{A_T^c}{L_T}\right] = \ln\frac{A_0}{L_0} + \left[-\sigma_L\lambda_L + \frac{\sigma_L^2}{2}\right]T.$$

In these equations, the symbols μ denote (instantaneous) expected excess returns, the λ are Sharpe ratios, the ρ are correlations and the σ are volatilities.

It is straightforward to see that the LUG neither depends on the initial funding ratio, nor on the Sharpe ratio of the liability process, since these two parameters vanish from the expression.

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deterministic function of time. This condition is satisfied by the optimal strategy of Proposition 2.1, since stochastic volatilities in the volatility matrix and the weight vector are cancelled out from the product. But the fixed-mix LDI strategy has constant weights, so that the vector $\underline{\sigma_t w}$ does depend on asset volatilities. In order to avoid having a stochastic volatility vector, we require constant asset volatilities.

Proposition 2.4 shows that the LUG of the fixed-mix strategy can be interpreted as the difference of two quadratic utilities, achieved respectively with the LDI strategy and the risk-free asset. Since the risk aversion γ is greater than 1, the coefficient of the variance is negative; as a result, the LUG is increasing in the correlations of both building blocks with liabilities. Hence, everything else equal, an investor will prefer the equity benchmark that has the largest correlation with liabilities.

As with the LUG associated with the optimal strategy, the LUG computed with the fixed-mix strategy can be used to compare two investment universes. Let us again consider the two investment universes we previously introduced, and define two LDI strategies as follows:

- For Strategy 0, the equity benchmark, S_0, is a broad CW market index, and the bond benchmark is the LHP, which has a correlation of 1 with liabilities and the same volatility as liabilities. We let x_0 denote the allocation to the benchmark S_0;

- For Strategy 1, the equity benchmark, S_1, is an portfolio constructed from a subset of the universe of the broad index in an attempt to enhance its liability-hedging properties. The bond benchmark is still assumed to be the perfect LHP, and the allocation to the benchmark S_1 is x_1.

As for the optimal strategy, the quantity of interest is the change in the LUG, $\Delta LUG^{fm} = LUG_1^{fm} - LUG_0^{fm}$. It is such that the investor is indifferent between investing A_0 in Strategy 1 and $A_0 \times e^{\Delta LUG^{fm}}$ in Strategy 0. By letting A_T^0 and A_T^1 be the terminal wealths obtained by investing A_0 in each of the two fixed-mix strategies, we obtain, from Proposition 2.4:

$$\Delta LUG^{fm} = E\left[\ln\frac{A_T^1}{A_0}\right] - E\left[\ln\frac{A_T^0}{A_0}\right] + \frac{1-\gamma}{2}\left\{V\left[\ln\frac{A_T^1}{L_T}\right] - V\left[\ln\frac{A_T^0}{L_T}\right]\right\}.$$

At this stage, the change in LUG not only depends on the risk and return parameters of the building blocks, but also on the unobservable risk aversion parameter, and the allocations to the respective equity benchmarks. To remove the dependency in the risk-aversion parameter, let us now assume that the allocation x_1 is chosen in such a way that the two funding ratios have the same variance. Then, the change in LUG reduces to the excess return of Strategy 1 over Strategy 0:

$$\Delta LUG^{fm} = E\left[\ln\frac{A_T^1}{A_0}\right] - E\left[\ln\frac{A_T^0}{A_0}\right].$$

The computation of the variance-matching allocation x_1 requires solving the quadratic equation $V\left[\ln\frac{A_T^1}{L_T}\right] = V\left[\ln\frac{A_T^0}{L_T}\right]$. This equation may have two, one or zero roots (in the field of real numbers). The discussion of the various cases is simplified by the assumption that the LHP is perfect. Indeed, the variance of the log funding ratio in this case simplifies to:

$$V\left[\ln\frac{A_T}{L_T}\right] = x^2[\sigma_S^2 + \sigma_L^2 - 2\sigma_S\sigma_L\rho_{S,L}]T = x^2 T E_S^2 T,$$

where TE_S is the annualised tracking error of the equity benchmark with respect to liabilities. Thus, excluding the negative root of the variance-matching equation, we have:

$$x_1 = \frac{TE_{S0}}{TE_{S1}} x_0.$$

In particular, the allocation to S_1 is higher than the allocation to S_0 as long as S_1 has lower tracking error. In other words, for a given risk budget (defined as the variance of the log terminal funding ratio), one can allocate more to the equity benchmark that has the lower tracking error. The corresponding change in LUG is given in the following proposition (proved in Appendix C).

Proposition 2.5. (**Change in LUG under Equal Variances**) *Consider two equity benchmarks S_0 and S_1, and one bond benchmark B that perfectly replicates liabilities. For each equity benchmark $X = S_0$ or $X = S_1$, let TE_X be the instantaneous tracking error with respect to liabilities, and $\mu_{X/L}$ be the expected return of the funding ratio X/L. For any initial allocation x_0 to the benchmark S_0, the change in LUG associated with the variance-matching allocation to the benchmark S_1 is given by:*

$$\Delta LUG = \left[\frac{TE_{S0}}{TE_{S1}} \mu_{S1/L} - \mu_{S0/L} \right] \times x_0 T.$$

ΔLUG is a linear function of x_0 and T, so the sign neither depends on the allocation to the benchmark S_0 nor on the investment horizon. This sign depends on the expected returns and volatilities of the benchmarks, expressed in relative terms with respect to liabilities: these parameters are themselves functions of the expected returns, volatilities and correlations with liabilities of the benchmarks.

For a given benchmark S_0, a given horizon T and a given allocation x_0, ΔLUG is a function of two variables, namely $(TE_{S1}, \mu_{S1/L})$. The indifference curve is the set of pairs such that ΔLUG is zero (i.e. such that the investor is indifferent between Strategies 0 and 1). It is described by:

$$\mu_{S1/L} = \frac{TE_{S1}}{TE_{S0}} \mu_{S0/L} = F(TE_{S1}).$$

If $\mu_{S1/L} > F(TE_{S1})$, that is if S_1 lies above the indifference curve in the $(TE_{S1}, \mu_{S1/L})$-diagram, then ΔLUG is positive. The opposite holds if $\mu_{S1/L} < F(TE_{S1})$. A graphical representation enables us to immediately visualise whether the replacement of S_0 by S_1 in the portfolio leads to a welfare improvement. At the end of Section 4, we will present examples of these indifference curves and their application to the selection of equity benchmarks leading to welfare improvements from an asset-liability management perspective.

The welfare change is unambiguously positive if S_1 dominates S_0 in the "relative mean-variance sense", that is, if the funding ratio $\frac{S_1}{L}$ has both lower tracking error and higher expected return than the funding ratio $\frac{S_0}{L}$. This makes sense: as explained above, lowering the tracking error of the equity benchmark allows one to allocate more to this building block without increasing the relative risk of the portfolio, and if the new benchmark has sufficiently higher expected return than the old one, the result is an increase in performance.

Overall, this analysis shows that welfare gains can be expected from decreasing the tracking error of the equity benchmark with respect to liabilities, unless this comes at too high a cost in terms of expected return. In the next sections of this paper, we conduct a thorough empirical study of the problem to construct equity benchmarks with reduced tracking error. Given the difficulty in forming reliable expected return estimates (Merton (1980)), the portfolio exercises that will be performed in Section 4 focus on achieving the highest possible degree of diversification, which is the most natural approach to reduce any excess of unrewarded risk from a given portfolio.

3 Introducing Formal Measures of Liability-Friendliness

Broadly speaking, there are two main ways of formally defining the notion of liability-friendliness.

- An economically-motivated definition based on *cash-flow matching*: The cash-flow matching approach to liability hedging aims at finding securities whose dividend or coupon payments match the liability payments as closely as possible, both in terms of size and schedule. In this context, a stock can be said to be more liability-friendly if it enjoys higher and more stable levels of dividend yield, that is dividends paid for each $100 invested.

- A statistically-motivated definition based on *factor exposure matching*: Since perfect cash-flow replication is typically difficult to achieve in practice, investors who need to hedge liabilities may instead choose to match the risk factor exposures of their assets with those of their liabilities. The objective pursued in this case is to immunise the funding ratio against variations in the risk factors that impact liabilities, and the success is measured in terms of tracking error with the liability proxy. In this approach, stocks with low tracking error with respect to liabilities should therefore be more attractive than the average in terms of liability-hedging purposes.

Obviously, these two perspectives are not mutually exclusive and we shall examine in Section 4 the impact of a joint selection procedure based on a combination of criteria focusing on an economically motivated criterion (such as dividend yield) and a statistically motivated criterion.

We now turn to a review of the related literature, which can accordingly be split into papers that have looked at the drivers of dividend yields and dividend yield stability, and papers that have looked at the relationship between stock return and risk factors impacting bond returns taken to be a proxy for liability returns.

3.1 Definition of Liability-Friendliness Based on Cash-Flow Matching

The purpose of this section is to characterise the stocks whose cash flows are the best at matching bond cash flows. We start by recalling that the one period total return of a stock can be decomposed in the following way:

$$R_{t,t+1} = \frac{S_{t+1} + D_{t,t+1}}{S_t} - 1 = \frac{S_{t+1}}{S_t} - 1 + \frac{D_{t,t+1}}{S_t},$$

where S_t is the time-t price of the stock and $D_{t,t+1}$ is the amount of dividend paid between t and $t+1$. The stock return can therefore be decomposed into two terms: the ex-dividend price return and the dividend yield. The principle of cash-flow matching requires that future pay-offs be as high as possible while having as small a variance as possible.

Fama and French (1988) report that over the 1926-1986 period, the mean annual price returns of value-weighted NYSE portfolio was 9.2% while their standard deviation was 20.6%. Over the same period, the mean of the corresponding dividend yields was 4.7% for a standard deviation of 1.2%. In other words, while the mean of dividend yields was equivalent to half of that of price return, their standard deviation was 17 times smaller, suggesting that dividends can provide a safer resource for matching fixed pension liabilities. Looking at aggregate S&P 500 data over 1935-2001, Ang and Liu (2007) report average returns of 12.5% with 16.9% standard deviations while the figures are 4.0% and 1.5% for dividend yields. The ratio between the standard deviations is equal to 11.26. Over the 1946-2006 period, Chen et al. (2012) also report standard deviations of S&P 500 dividend yields of 1%, but they do not provide the corresponding volatility of stock returns (which we estimated to be 14.1% using the monthly returns of Robert Shiller's stock market data - available at http://www.econ.yale.edu/s̃hiller/data.htm). All of these figures show that price returns are 10 to 20 times more volatile than dividend yields while they are only 2 to 4 times larger on average. This is one motivation for focusing on the dividend yield portion of the total return which is much less subject to large variations and hence more reliable for cash-flow matching purposes. Another motivation for selecting stocks paying a high dividend yield is that the use of dividends to meet pension payments reduces the turnover of the portfolio since it reduces the amount that would have to be liquidated to match the cash-flows.

More formally, we have the following variance decomposition:

$$V(R_{t,t+1}) = V\left(\frac{S_{t+1}}{S_t}\right) + V\left(\frac{D_{t,t+1}}{S_t}\right) + 2\,Cov\left(\frac{S_{t+1}}{S_t}, \frac{D_{t,t+1}}{S_t}\right),$$

where the third term is usually negligible because the correlation between price returns and dividend yields is close to zero (this stems from the fact that dividend yields are always positive while the sign of returns alternates and the terms in the covariance often cancel out). In the next section, the case will be made for the reduction of the whole return variance, in order to lower the tracking error. In this section, we focus on the second term, which is the one that can be lowered to very small levels (1% or 2% standard deviation) while being associated to strictly positive cash-flows.

The average dividend yield is straightforward to compute. It is the primary variable for the size of the cash-flow arising from dividend payments. In our framework, high average dividend yields must be combined with low variance of dividend yields. At the aggregate level, there is no technical hurdle

in the computation of dividend yield variability. At the individual stock level, on the other hand, problems arise. First, we want to be able to compare stocks in the cross-section, and a firm with high average dividend yield will mechanically have a higher dividend yield variance. Normalisation procedures (dividing times series by their mean for instance) do not provide satisfactory answers to this problem because a constantly increasing series will be penalised compared to a series oscillating around its mean (and furthermore, some series have a zero average and cannot be normalised). One better solution is to look at the variance of dividend yield *growth* (i.e. dividend yield return). Not only does this eliminate the size effect, but it also addresses possible stationarity issues which may arise when working with raw dividend yield series. Unfortunately, the computation of the variability of dividend growth is only possible for strictly positive series (when firms pay a dividend each year without any exception) which are in fact rare in data samples.

From a technical standpoint, it is consequently hard to estimate dividend yield volatility at the individual firm level, because companies may cease to pay dividends for some periods of time. This creates outliers in the dividend yield series, and perturbs the computation of volatility. In view of this technical difficulty, it would be of interest to have proxies for the uncertainty in dividend yield or dividend yield growth, either observable or easier to estimate than the volatility of the dividend yield process. This exercise, to which we turn after a brief review of the related literature, is the objective of Section 3.1.2.

3.1.1 Review of the Related Literature

A number of articles have analysed the relationship between dividend yields and stock returns. For instance, using a specific parametric model (where dividend yields are mean-reverting through a square-root process), Ang and Liu (2007) show that the conditional drift of stock returns is increasing in the dividend yield, while the conditional volatility of returns decreases with dividend yields.

In a seminal article, Campbell and Shiller (1988) provide an approximate representation of the log dividend yield as the expectation of a linear combination of future discount rates and future dividend growth. The variance of the dividend yield can then be decomposed into a component that stems from the future discount rates and one that can be attributed to future dividend growth. While Cochrane (2011) finds that the larger of the two is the first one for the aggregate market, Maio and Santa-Clara (2014) show that there are in fact large cross-sectional differences and that future dividend growth is more important when looking at portfolios of value or small cap stocks.

An alternative way of addressing the question is to consider the following second order Taylor approximation:

$$V\left(\frac{D}{S}\right) \approx \frac{V(D)}{E(S)^2} - 2\frac{E(D)}{E(S)^3}Cov(S, D) + \frac{E(D)^2}{E(S)^4}V(S),$$

where S denotes the stock price and D the dividend.

If the variance of S is not too large,[5] then dividend yield variance is increasing in dividend variance. Accordingly, an analysis of the literature on dividend variability may provide useful insights.

From a theoretical standpoint, dividends and stock prices are linked by the *discounted cash-flow* (or *present-value*) model. In its most general form, the price of the stock at time t is given by

$$S_t = E_t \left[\sum_{k=1}^{\infty} M_{t,t+k} D_{t+k} \right],$$

where E_t is the expectation conditional to the information available at time t and $M_{t,t+k} = \prod_{j=0}^{k} M_{t+j}$ is the product of a stochastic discount factor between date t and $t+k$. The classical Gordon model is obtained when the discount factors are constant through time($M_{t+j} = (1+r)^{-1}$) and when $D_{t+k} = D_t(1+g)^k$ (i.e. the growth of dividend is also assumed constant in time), with $g < r$.

In order to better understand the determinants of dividend decisions at the firm level, we now turn to the literature which focuses on the drivers of corporate dividend policy. In their article on the characterisation of firms which pay out, Fama and French (2001) find that "*dividend payers tend to be large, profitable firms*". However, even after controlling for characteristics such as size, book-to-market or profitability, they observe a decline in the propensity to pay dividends, from the 1970s to the late 1990s. Hoberg and Prabhala (2009), in addition to the factors proposed in Fama and French (2001), study the impact of risk variables. They perform logit regressions (1 if firms pay dividends and 0 if not) on idiosyncratic and systematic risk variables. Their results show that the propensity to pay dividends is negatively exposed to both sources of risk after controlling for the Fama-French factors. These negative exposures are strongly statistically significant. They report that "*risk is a significant determinant of the propensity to pay dividends, and it explains roughly 40% of disappearing dividends*". Consequently, a firm is more likely to maintain its dividend payments if it is less risky.

Coming back to the issue of cash-flow stability, we examine the cross-sectional determinants of dividend variability through the lens of dividend smoothing. From an accounting standpoint, dividends are by nature essentially driven by earnings. Lintner (1956) proposes the following model of a dividend adjustment based on earnings (denoted as E_t):

$$\Delta D_t = \alpha_0 + \alpha_1 E_t + \alpha_2 D_{t-1} + \epsilon_t,$$

where $-\alpha_1/\alpha_2$ is the *target payout ratio* (TPR), that is, the proportion of earnings that the company wishes to distribute as dividends. This model is meant to reflect the fact that firms, when facing a strong increase in earnings, do not raise their dividend so as to maintain a constant TPR. Instead, they cap the increase in dividends in order to avoid having to cut them in the event of a future negative shock on earnings. In doing so, they smooth their dividend payments. This can be measured by the *speed of adjustment* (SOA), equal to $-\alpha_2$: a firm that smooths its dividend has a SOA close

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[5]More precisely, if $\sigma_S < \sigma_D \rho_{D,S} E(S)/E(D)$, where $\rho_{D,S}$ is the correlation between the two variables. This condition is realistic when looking at short to middle term horizons (2-10 years) or when considering low volatility stocks.

to zero. Alternative practical definitions of dividend smoothing can be found in Chen et al. (2012) or in Leary and Michaely (2011). In the latter reference, the authors show that young, small firms, with low dividend yield and high volatility of returns, smooth less than their older and less volatile counterparts. They divide their dataset into quintiles of SOA and show that the SOA and stock volatility are positively correlated (i.e. that the lowest average volatility is associated to the first SOA quintile while the highest average volatility corresponds to the last SOA quintile). This relationship is monotonic over the five quintiles. Similarly, they show that the relationship is reversed when looking at dividend yields: the firms that are the most likely to smooth are those with the highest dividend yields. Overall, the literature on dividend variability points out the same favourable characteristics as those mentioned by Baker and Wurgler (2012) for bond-like stocks: low volatility, high dividend yield and large size. These features sound intuitive, as they point towards mature, dividend paying, companies.

Regardless of the specific model that is assumed, and regardless of the drivers of dividend variability, a broad intuition conveyed by the discount dividend model is that stock return volatility should be increasing in the volatility of future dividends. A large number of papers have actually proposed theoretical models which lead to an explicit (and often affine) relationship between dividend uncertainty and stock volatility. For instance, in Campbell et al. (1997), the variance of future expected returns is equal to the variance of news about future dividends plus an additional term. Furthermore, the model proposed by Wu (2001) implies that the conditional variance of future returns is proportional to the conditional variance of dividend growth. Another example is the aggregate model of Bansal and Lundblad (2002), in which the market conditional volatility is an affine function of the market cash flow volatility. Further references of such relationships are provided by Spiegel (1998) and Li and Yang (2013). Other similar relationships are obtained for the variance of the price-dividend ratio in Cochrane (1992) or the variance of the log dividend-price ratio in Ang (2012). A common conclusion of these articles is that stock volatility is increasing in dividend or dividend growth volatility, with the key implication that low volatility stocks are likely to be stocks with stable dividend streams.

3.1.2 Empirical Characterisation of Stocks with Higher Cash-Flow Matching Capacity

In order to confirm the insights from the literature, we want to investigate the link between dividend variability and other stocks' attributes in our dataset. Following the results of Baker and Wurgler (2012), we empirically analyse various selection criteria based on volatility, dividend yield and capitalisation, and we measure the impact in terms of out-of-sample average dividend yield and dividend yield volatility.

As explained above, this analysis would be difficult to carry out at the stock level: indeed, stocks pay dividends only a few times in a year, and the payment dates for a given firm may vary from one year to another. This results in a series with low frequency and irregularly spaced observations. Moreover, it often happens that a stock does not pay dividends in a given year: thus, the dividend

yield series may jump to zero from one year to another. Clearly, the sample volatility of such a series would not be reliable (and the standard deviation of dividend yield growth in this case is not even finite when dividends fall to zero). In order to have a series with smoother growth, we aggregate stocks in portfolios: in this case, the series are strictly positive and the dividend yield growth is well defined.

Our data is extracted from the CRSP/Compustat database, and covers the 1946-2013 period. We filter stocks in order to exclude outliers[6] in returns and dividend growth, and to retain only the firms traded for at least 30 years and which have had at least 15 years of an active dividend policy. This narrows the universe to 2,623 stocks. Each year t between 1956 and 2012, we split the stocks into deciles based on the volatility of their annual returns over the past ten years. We accordingly build ten EW portfolios, ranging from the one which is expected to be the least volatile to the one which bears the largest ex-ante individual risks. We average the corresponding dividend yields over the following year for each portfolio and perform a new split at the end of the year. At the end of the sample, we have ten annual time series and we compute the average and the volatility of each series (that is, the standard deviation of the returns of the series). Moreover, we provide the out-of-sample volatility (standard deviation of annual returns) of each portfolio. The dividends are extracted from the Compustat database (item "Dividends Common/Ordinary"), the dividend yield is computed as the ratio of Compustat item "Dividends Common/Ordinary" paid between year $t-1$ and year t and Compustat item "Capitalization" at year $t-1$. In addition, we perform the same analysis but building portfolios on deciles of dividend yields and capitalisation. These criteria are averaged over the past 10 years. The results are summarised in Table 1. In order to compare the significances of the relationships between the standard deviation of time series and the tested criteria, one can use the HAC p-value proposed in the comparison test of Ledoit and Wolf (2011). We compare the first and last times series, that is the portfolios based on the extreme deciles. The last column of the table indicates that the difference in dividend yield volatility between the extreme deciles is indeed significant.

The portfolios sorted on volatility display the expected pattern: the standard deviation of the growth rates of the dividend yield is increasing in stock volatility, which confirms the link between stock return volatility and volatility of dividend growth (see also the predictions of an explicit asset pricing model in Section 3.2.3 in case of low or moderate levels of correlation between the interest rate and the dividend process). The monotonicity is almost strict even though the second-to-last decile breaks the increasing pattern. It is thus found that low stock volatility signals low uncertainty for dividend growth, which is one attribute of bond-like liability-friendliness. Moreover, the average dividend yield (Panel A) is strictly decreasing with stock volatility (it is the opposite for out-of sample volatility in Panel C). Therefore, low volatility stocks will have two favourable features for cash-flow matching, namely higher average dividend flows and lower variability of these flows.

The exact same conclusions hold for the selections based on dividend yields, but the magnitudes are even larger: the spread in average dividend yield between extreme deciles is equal to 4.65% for the

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[6]An observation is characterised as an outlier if it is larger than the mean of the sample plus ten standard deviations of the sample or if it is smaller than the mean minus ten standard deviations of the sample.

Criterion	Deciles										p(10-1)
	1	2	3	4	5	6	7	8	9	10	
Panel A: Average dividend yield of portfolios(%)											
Volatility	7.51	5.91	5.20	4.56	4.39	4.24	3.89	3.42	3.22	2.86	
Dividend Yield	1.23	2.14	2.61	3.05	3.53	3.93	4.40	5.11	6.55	12.3	
Capitalisation	4.11	4.55	4.65	4.38	4.68	4.96	4.61	4.33	4.53	4.21	
Panel B: Standard deviation of dividend yield growth (%)											
Volatility	16.5	16.7	18.0	19.9	20.7	22.3	23.8	31.7	27.6	33.6	<0.01
Dividend Yield	34.8	26.2	21.8	22.0	19.3	20.0	18.5	16.7	18.6	16.0	<0.01
Capitalisation	23.8	26.8	26.0	16.6	20.8	19.7	22.8	17.3	19.3	15.0	<0.01
Panel C: Volatility of portfolios (%)											
Volatility	13.7	14.0	14.5	15.9	16.4	16.3	18.6	19.8	21.3	24.2	<0.01
Dividend Yield	21.1	17.9	17.3	18.5	17.9	17.1	15.6	17.0	15.6	16.2	<0.01
Capitalisation	25.2	21.8	20.3	19.0	19.0	17.6	17.9	17.0	16.7	14.4	<0.01

Table 1: **Average dividend yields and dividend yield volatility**. The sample comes from the annual Compustat / CRSP database over the period 1946-2012. Firms with less than 30 years of data and 15 years of active dividend policy are excluded. At the end of each year t, the firms are sorted in deciles on one criterion (volatility, dividend yield, capitalization) computed over the past 10 years. The average dividend yields corresponding to each decile are then computed based on their value of year $t + 1$. The procedure is repeated until the end of the sample. The table reports the average dividend yield and the standard deviation of the returns of the dividend yield of each decile. The last column provides the HAC test p-value detailed in for the difference between the variances of the first versus last decile.

volatility selections, but it reaches 11.1% for the dividend yield selections. Likewise, for the volatility of dividend yields, the spread is 17.1% for the volatility selections and 18.8% for the dividend yield selections. However, the pattern is less pronounced for the out-of-sample volatility of the portfolios (Panel C). These results are in line with the findings of Leary and Michaely (2011), who show that high dividend yield and low risk firms tend to smooth their dividend policies more than their risky, low dividend counterparts. Accordingly, a selection of high dividend yield stocks is expected to potentially be as relevant as a selection of low volatility stocks for cash-flow matching purposes.

On the other hand, the results for the portfolios based on the capitalisation do not show a clear, monotonous impact on average dividend yield, or on the standard deviation of dividend yield growth, even though they suggest that large cap firms show significantly lower dividend yield uncertainty compared to the smallest cap firms. Panel C is an illustration of the the well-known stylised fact that small cap firms are more volatile than their large cap counterparts. While it is more difficult to discriminate on this criterion within either the small or the large cap universe for the purpose of constructing portfolios with enhanced cash-flow matching capability, one insight we obtain is that larger cap stocks should be broadly preferred to smaller cap stocks. For this reason, and also because

of the higher liquidity on large cap stocks (and longer available time-series), we shall restrict our analysis to the S&P 500 universe in Section 4.

3.2   Definition of Liability-Friendliness Based on Factor Exposure Matching

The second approach to the selection of stocks with enhanced liability-friendliness requires the use of specific criteria to assess the statistical proximity between stock returns and factors impacting liability returns. From the analysis in Section 2, it appears that the most straightforward indicator is the tracking error (TE) with respect to liability returns, defined as the standard deviation of stock returns in excess of liability returns. Given that in this paper we assume the existence of a bond index with the relevant duration that can be taken as a perfect proxy for the liabilities, this measure of tracking errors of stocks with respect to liabilities can be written as a function of the stock and bond volatilities (σ_S and σ_B), and the correlation $\rho_{S,B}$ between the two assets:

$$TE_{S,L} = \sqrt{\sigma_S^2 + \sigma_B^2 - 2\sigma_S\sigma_B\rho_{S,B}}.$$

In particular, we find that the tracking error is decreasing in the stock-bond correlation ($\rho_{S,B}$). Thus, everything else equal, selecting stocks that deliver a high correlation with bonds leads to a reduction in the tracking error. The tracking error also depends on the stock volatility (σ_S): strictly speaking, the impact is not monotonic since the tracking error is the square root of a quadratic function of σ_S. In detail, the derivative of the tracking error with respect to σ_S is given by:

$$\frac{\partial TE_S}{\partial \sigma_S} = \frac{1}{TE_S} \times 2(\sigma_S - \sigma_B\rho_{S,B}),$$

a quantity that is positive only if $\sigma_S > \sigma_B\rho_{S,B}$. But as long as stock volatility is larger than bond volatility, a condition which is most often satisfied in practice, the condition $\sigma_S > \sigma_B\rho_{S,B}$ is satisfied (since the correlation is at most equal to 1). Hence, the tracking error is increasing in stock volatility, so reducing tracking error also calls for choosing stocks with low volatility. Eventually, liability-friendly stocks are to be searched among those stocks that have low tracking error with respect to bonds, coming either from a high correlation with bonds, or low volatility, or ideally both. We note that the relative importance of the two dimensions depends on the duration of the liabilities, with an expected domination of volatility reduction for short duration liabilities.

Before formalising these intuitions in the context of a formal asset pricing model, we first review the literature that has analysed the relationship between stock returns and bond returns, and the related literature that has focused on the relationship between stock returns and changes in interest rates.

3.2.1 Cross-Sectional Differences in Stock-Bond Comovements

There is a large consensus that the correlation between stock and bond returns is strongly time varying. At the level of an aggregate stock index, it has ranged between -60% and +60% over the past few decades, and has experienced variations of $\pm 20\%$ from one month to the other (see Baele et al. (2010)).[7] This variability is exacerbated at high frequencies, as shown in Aslanidis and Christiansen (2012). Another finding is that the correlation is linked to market conditions, being generally lower in "bad" times. For instance, Andersson et al. (2008) report results "*consistent with the 'flight to quality' phenomenon*" which "*suggest that periods of elevated stock market uncertainty lead to a decoupling between stock and bond prices*". In the US market, Yang et al. (2009) find that correlations are lower in recessions than in expansion periods. Thus, for stock indices, there seems to be a negative relationship in time series between volatility and bond correlation. Connolly et al. (2005) also show that the VIX level has predictive power with respect to the sign of the correlation: periods of high uncertainty tend to be followed by negative correlations.

At the individual stock level, Baker and Wurgler (2012) study the impact of various attributes on the "bond-beta" of stocks: this beta is estimated by regressing stock excess returns against the market factor and a bond excess return. The inclusion of the market factor is standard in such regressions, and aims at controlling for the effect of the dominant equity factor.

Among the characteristics considered by Baker and Wurgler (2012), some are observable, such as size, dividend yield and book-to-market ratio, and the last one, namely volatility, is not. Interestingly, the negative link between volatility and correlation observed in time series at the index level is also found to exist in the cross-section: when stocks are sorted on volatility, the bond beta varies monotonically across volatility deciles, the high volatility decile displaying the lowest value. Moreover, the bond beta is positive only for the highest two volatility deciles. In other words, it is the stocks with the lowest volatilities that co-move the most strongly with bonds. Among the other tested characteristics, only size (measured by market capitalisation) also gives rise to a monotonic pattern across deciles: larger cap stocks have higher bond betas than smaller cap stocks, an effect which can partially be related to the low volatility effect since large cap stocks tend to have lower volatility on average compared to small cap stocks. The dividend-to-book ratio implies a quasi-monotonic pattern: firms with high ratio tend to be more bond-like than the others. The age of the firm (as measured by the time for which it has been listed in the CRSP database) also appears to be a potentially useful characteristic: old firms tend to be more bond-like than the young ones. But the effect is not as pronounced as it is for volatility, and, more importantly, the bond exposures are negative for all age deciles, while deciles of stocks with low volatility, high dividend-to-book or large size display positive betas. The book-to-market ratio (B/M) generates a U-shaped pattern: firms with extreme ratios are less bond-like than those that lie in the middle. To the extent that very high ratios actually signal

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[7]Several models have been developed to capture the time variation in the stock-bond correlation. See for instance Cappiello et al. (2006), Guidolin and Timmermann (2006), Bekaert et al. (2010), Baele et al. (2010), Wu and Liang (2011), and David and Veronesi (2013).

distressed firms, this fact suggests that both growth firms and distressed firms are less bond-like than those that have average ratios. All of these effects are observed both in the full sample and in "de-coupling periods", which are periods where the aggregate market stock index covaries negatively with the bond index.

Eventually, Baker and Wurgler (2012) find that the stocks with positive bond beta are those that belong to one of these categories: large capitalisation; low volatility; or high dividend-to-book ratio. It is remarkable that low volatility proves to be associated with high bond beta, while it is also desirable for the purpose of narrowing the gap between stock and bond volatilities.

3.2.2 Cross-Sectional Differences in Equity Duration

As bond returns have a strong negative correlation with interest rate changes, the search for a high stock-bond correlation is almost equivalent to the search for a strongly negative stock-interest rate correlation (the two objectives would be mathematically equivalent if the correlation between bonds and interest rates was exactly -1). In other words, one should be looking for stocks whose prices tend to respond to interest rate movements in the same way as bond prices. Thus, if one has a measure of "equity duration", defined as the sensitivity of a stock price with respect to interest rate variations, a possible criterion for the selection of bond-like stocks is to retain stocks with positive duration. We now review different methods for evaluating equity duration.

The notion of equity duration is meant as a quantitative measure of the link between stock prices and interest rates. It provides a criterion to eliminate non bond-like stocks: indeed, a fixed-income bond always has positive duration, so to be bond-like, a stock must have positive duration as well, which excludes stocks with negative duration. Equity duration can also serve as a tool to construct an equity portfolio that matches the duration of a given bond.

The literature on this topic can be divided in two strands:

- An analytical approach, which defines duration as the sensitivity to changes in an interest rate proxy and relates it to other stock's characteristics;

- A statistical approach, which treats duration as an interest rate beta and estimates it by regression techniques.

Analytical approach   The analytical approach defines equity duration as the sensitivity to interest rate changes, as is done for fixed-income securities. In the Dividend-Discount Model (DDM), the price of a stock is viewed as the sum of cash flows discounted at a uniform rate k, the cash flow stream having an infinite maturity (since stocks, unlike bonds, do not have a pre-determined lifetime). We let δ_t be the dividend paid in period $[t-1, t]$, and k be the discount rate, so the current stock price is:

$$S_0 = \sum_{t=1}^{\infty} \frac{\delta_t}{(1+k)^t}.$$

As for bonds, one can define "modified" and "Macaulay" durations: [8]

$$D_0^{k,mod} = -\frac{1}{S_0}\frac{\partial S_0}{\partial k}, D_0^{k,mac} = \sum_{t=1}^{\infty} t \times \frac{\delta_t}{(1+k)^t} = (1+k)D_0^{k,mod}.$$

The Gordon model (Gordon (1982)) assumes that dividends grow at a constant rate g, which implies that modified duration is the reciprocal of the dividend yield (DY):

$$D_0^{k,mod} = \frac{S_0}{\delta_1}.$$

Hence, high DY stocks have shorter durations than low DY ones. This property is natural given the model's assumptions: stocks that pay high dividends today, derive most of their value from short-term cash flows, and thus have a shorter average life, exactly as bonds that pay a high coupon.

The assumption of a constant dividend growth rate is clearly restrictive. Dechow et al. (2004) relax it by introducing an alternative model for cash flow evolution until a forecasting horizon (which they take equal to 10 years). Dividends are constant after the horizon in question. However, the model still assumes a deterministic evolution. Under specific assumptions (including notably zero growth in equity), the authors are able to relate Macaulay duration to standard financial ratios: duration can be expressed as an increasing function of the price-earnings ratio, and as a decreasing function of the book-to-market ratio.

The assumption of deterministic cash flows is hardly appropriate for stocks, especially for those that have the opportunity to start new business in the future. Thus, Leibowitz and Kogelman (1993) introduce a " Franchise Factor Model" that makes an explicit distinction between the "tangible value", derived from the continuation of current business, and "franchise value", which comes from future business opportunities. Assets in place typically have a long duration, but the cash flows generated by future projects can vary with interest rates, which makes their duration more difficult to predict. Hevert et al. (1998) note that if growth options were analogous to European call options, they would contribute negatively to duration, since the value of a call is increasing in the interest rate. On the other hand, growth options do not have a fixed exercise price, so their effect on duration is unclear. Overall, one expects that firms that derive most of their value from assets in place will have higher durations than those with growth options.

Another issue raised by the standard DDM is that it treats the discount rate and the cash flows as independent variables, thereby ignoring the " flow-through" effect, namely the joint impact of an interest rate shock on both variables. Leibowitz et al. (1989) argue that this leads to overestimating equity duration, and they introduce an adjustment to the DDM duration that corrects for the impact of interest rates on cash flows. Qualitatively, the flow-through effect will be higher for firms whose cash flows increase when interest rates increase than for firms whose cash flows are relatively insensitive to

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[8]The two definitions would coincide if the discount rate was continuously compounded (see the appendix in Leibowitz et al. (1989)).

interest rates, and the latter stocks will have higher durations. In particular, sectors with low pricing power, such as utilities, are expected to fall in this category.

The DDM is not the only model that allows for the derivation of equity duration. As explained in Fishwick (2013), the Capital Asset Pricing Model of Sharpe (1964) can also be used for this purpose. Indeed, under the CAPM's assumptions, the conditional expected return on a stock is a function of its market beta:

$$E_t \left[\frac{S_{t+1}}{S_t} - 1 \right] - R_{ft} = \beta (M_t - R_{ft}),$$

where M_t is the expected return on the market portfolio and R_{ft} is the one-period risk-free interest rate. Rearranging terms, we obtain:

$$S_t = \frac{E_t[S_{t+1}]}{1 + \beta M_t + (1 - \beta) R_{ft}}.$$

Hence, stocks with $\beta > 1$ have a price increasing with the risk-free rate, and the opposite holds for $\beta < 1$. In other words, stocks with $\beta < 1$ are like bonds, while the others are not. This analysis again calls for the selection of low-beta stocks, and suggests that the reported outperformance of low volatility stocks (the so-called low volatility anomaly of Ang et al. (2006, 2009)) can be explained by a stronger exposure to interest rate changes over a sample period where interest rates have been mostly decreasing. A similar criterion can be found in Casabona et al. (1984), who show in the context of the DDM that equity duration is decreasing in the discount rate k, and that the CAPM implies that k is increasing in the market beta. Hence, duration is decreasing in the market beta. This theoretical prediction is of course consistent with the empirical finding of Baker and Wurgler (2012) that low volatility stocks (which tend to have low betas) are more bond-like compared to other stocks.

Statistical Approach The general principle is to regress the realised return on an interest rate factor, with possible inclusion of additional control variables:

$$r_{St} = \alpha + \beta_r F_{rt} + \beta_{c1} F_{c1,t} + \cdots + \beta_{cn} F_{cn,t} + \epsilon_t,$$

where r_{St} is the realised stock return in period $[t-1, t]$, F_{rt} is the interest rate factor and $F_{c1,t}$, ..., $F_{cn,t}$ are n control variables.

As far as the interest rate factor is concerned, the most common option is to take the change in a long-term sovereign interest rate and equity duration is then identified with $(-\beta_r)$. This choice is made in Sweeney and Warga (1986), Hevert et al. (1998) and Reilly et al. (2007). The identification of $(-\beta_r)$ with equity duration is justified by the fact that for a bond index, the above regression returns an interest rate beta which is close to the negative of modified duration (see Reilly et al. (2007)). Cornell (2000) makes a different choice of regressor, advocating the use of the return on a Treasury bond in order to have an exact matching of dates for returns on both sides of the regression equation, in which case duration is directly measured by β_r. He also argues that since bond returns are strongly

negatively correlated with yield changes, the durations obtained by this method should be close to proportional to those obtained with an interest rate change.

Regarding the inclusion of other control variables, since the market is the dominant factor in equity returns, it is likely that a univariate regression on F_{rt} would only reflect differences in the market exposures (see Cornell (2000) for a discussion). This motivates the introduction of the market factor as a control variable, a specification taken in Hevert et al. (1998), Cornell (2000), Reilly et al. (2007).[9] Sweeney and Warga (1986) adopt a different form, where the market factor is first orthogonalised against F_{rt}. While it can be shown that this leaves the interest rate beta and its t-statistics unchanged, this second specification is better suited to find whether the interest rate factor in itself is rewarded (i.e. if it carries an independent risk premium above and beyond the premium arising from the correlation with the market). Using the bivariate analysis, Hevert et al. (1998) find that low B/M portfolios have lower durations than high B/M ones, but this conclusion is questioned by Cornell (2000), who shows that the effect is not monotonic across quintiles of B/M. In fact, his results suggest a U-shaped impact of B/M, which is reminiscent of the effect documented by Baker and Wurgler (2012) for the stock-bond correlation: duration is higher for middle quintiles of B/M than in the extreme classes. The regression approach also enables a size effect to be recovered, as in Baker and Wurgler (2012): Cornell (2000) and Reilly et al. (2007) find that duration is increasing in market capitalisation. The size effect almost disappears when all three Fama-French factors are introduced as control variables. Cornell (2000) attributes this to the significant statistical relationship between the interest rate and the size factors. Reilly et al. (2007) also find differences across sectors: technology and electronic equipments, which have more pricing power than utilities and food, also have lower, and sometimes negative, durations. They interpret this sector effect as the result of different pricing powers: in a period of surging inflation, technological firms can raise their prices more easily than their utility sector counterparts, because they produce high added-value goods, for which no substitute is available on the market. Thus, if an interest rate rise follows from a positive inflation shock, the cash flows of the former firms will react more than those of the latter, which will tend to offset the impact on the discount rate. Hence, it is expected that stocks that belong to sectors with low pricing power will display higher durations.

From the time series perspective, the statistical estimate for equity duration shares at least one property with the stock-bond correlation: it is a highly time-varying measure. Reilly et al. (2007) find that the duration of a given stock index estimated over a rolling window can even change sign over time. On the other hand, the duration of a bond index is always positive and in any case much more stable over time. From a liability hedging standpoint, the instability of equity duration makes it difficult to construct a duration-matching portfolio based on this measure.

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[9]The bivariate regression of stock returns on bond returns and market factor is also the specification taken in Baker and Wurgler (2012), although they do not call the resulting bond beta the duration of the stock.

3.2.3 Predictions from an Asset Pricing Model

So as to better analyse the drivers of stock returns and their relationship with liability returns, we now look at the characteristics of bond-like stocks in the context of a formal asset pricing model. To this end, we introduce a continuous-time model, where stocks are represented as claims on uncertain dividends, and bonds are claims on a stream of fixed coupons.

State Variables and Asset Prices   As in Section 2, uncertainty is represented by a probability space (X, F, P) equipped with a two-dimensional Brownian motion $(\underline{z}_t)_{t \geq 0}$ and equipped with the filtration $(F_t)_{t \geq 0}$ generated by this process. The time span is $[0, \infty[$. It is infinite because we will consider securities that make perpetual payments.

We again assume that the short-term interest rate follows the Vasicek model Vasicek (1977):

$$dr_t = a(b - r_t)dt + \underline{\sigma}'_r d\underline{z}_t,$$

where $\underline{\sigma}_r$ denotes the 2×1 volatility vector. With a constant price of interest rate risk, this model implies that the price of a zero-coupon bond paying \$1 at date t_i is an exponential affine function of the short-term rate:

$$p(t, t_i) = \exp[-D(t_i - t)r_t + E(t_i - t)],$$

where the duration and the constant term are given by:

$$D(s) = \frac{1 - \exp[-as]}{a},$$

$$E(s) = b[D(s) - s] + \frac{\sigma_r^2}{2a^2}\left[s - 2D(s) + \frac{1 - \exp[-2as]}{2a}\right],$$

$$\tilde{b} = b - \frac{\sigma_r \lambda_r}{a}.$$

The parameter $\tilde{b}$ is the long-term mean adjusted for the price of interest rate risk.

We consider for simplicity a perpetual bond that pays annual coupons, all equal to C: the annual dates (end of calendar year) are denoted with t_1, t_2, ..., t_i, ... In the absence of arbitrage opportunities, the bond price is the sum of the discounted coupon payments:

$$B_t = C \sum_{\substack{i=1 \\ t_i > t}}^{\infty} p(t, t_i).$$

Note that with this formula, the bond price is right-continuous and discontinuous on the coupon dates, the price dropping by an amount equal to the coupon payment:

$$B_{t_i} - B_{t_i-} = -C.$$

From a technical perspective, it is desirable to exclude infinite values for the infinite sum that defines the bond price. To find out which combinations of parameter values avoid such infinite values, we use the D'Alembert criterion. It can be shown that the ratio $\frac{p(t,t_{i+1})}{p(t,t_i)}$ shrinks to zero if, and only if:[10]

$$\tilde{b} - \frac{\sigma_r^2}{2a^2} > 0.$$

We will thus restrict our attention to parameter values that satisfy this condition.

The stock is a claim on an infinite dividend stream. For simplicity, we assume that the dividend payment dates coincide with the coupon dates, and that the dividends are sampled from a Geometric Brownian motion:

$$\frac{d\delta_t}{\delta_t} = \mu_\delta dt + \underline{\sigma}'_\delta d\underline{z}_t.$$

The stock price is thus the present value of the dividend stream $\delta_{t_1}, \ldots, \delta_{t_i}, \ldots$ In order to price the dividend payments, we assume the existence of a constant price of risk for dividend risk, λ_δ, and we let $\underline{\lambda}$ denote the price of risk vector, defined by:

$$\underline{\lambda} = \underline{\sigma}(\underline{\sigma}'\underline{\sigma})^{-1} \begin{pmatrix} \sigma_r \lambda_r \\ \sigma_\delta \lambda_\delta \end{pmatrix}.$$

Thus, the pricing kernel $(M_t)_{t \geq 0}$ is:

$$M_t = \exp\left[-\int_0^t \left(r_s + \frac{\|\underline{\lambda}\|^2}{2} \right) ds - \int_0^t \underline{\lambda}' d\underline{z}_s \right].$$

We recall that the defining property of a pricing kernel is that the value of any self-financing portfolio divided by M_t follows a martingale. With these notations, the price of the payoff δ_{t_i} at date t is the conditional expectation of $\left(\frac{M_{t_i}}{M_t} \delta_{t_i} \right)$. Hence, the stock price is:

$$S_t = \delta_t \sum_{\substack{i=1 \\ t_i > t}}^{\infty} E_t \left[\frac{M_{t_i}}{M_t} \delta_{t_i} \right].$$

After some algebra, we arrive at the following expression:

$$S_t = \delta_t \sum_{\substack{i=1 \\ t_i > t}}^{\infty} \exp\left[(\mu_\delta - \sigma_\delta \lambda_\delta)(t_i - t) - \frac{\sigma_r \sigma_\delta \rho_{r\delta}}{a}[t_i - t - D(t_i - t)] \right] p(t, t_i).$$

Of course, we recover the expression for the bond price by taking a constant dividend process. More generally, as the bond price, the stock price is right-continuous, and the discontinuities occur

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[10]Using the D'Alembert criterion, the bond price is infinite if $\tilde{b} - \frac{\sigma_r^2}{2a^2}$ is negative, and the criterion is inconclusive as to the convergence of the infinite series when this quantity is zero.

on the dividend payment dates. Moreover, an application of the D'Alembert criterion shows that a sufficient condition for the stock price to be finite is:

$$\mu_\delta - \sigma_\delta \lambda_\delta - \frac{\sigma_r \sigma_\delta \rho_{r\delta}}{a} < b - \frac{\sigma_r^2}{2a^2}.$$

The stock price in our model has a similar (affine) form as the one in Lettau and Wachter (2007). The main difference between the two approaches is that we consider the dividends and interest rates as our two state variables while consider the dividends plus an additional non-observable state variable which drives the stochastic discount factor.

Model-Implied Bond and Stock Moments The bond and stock prices can be expressed as functions of time and the state variables r and δ:

$$S_t = S(t, r_t, \delta_t), B_t = B(t, r_t).$$

The dynamics of these two prices can be obtained by applying Ito's lemma. To save space, we only write down the expressions for the stock: those for the bond follow by setting the dividend growth rate and volatility equal to zero, and the dividend value equal to the coupon payment. We have:

$$\frac{dS_t}{S_t} = [\mu_{S,t} - \delta_{t_i} \times 1_{\{t=t_i\}}]dt + \underline{\sigma}'_{S,t}d\underline{z}_t,$$

where the volatility vector and the ex-dividend expected return are given by:

$$\underline{\sigma}_{S,t} = \underline{\sigma}_\delta + \frac{1}{S_t}\frac{\partial S}{\partial r_t}\underline{\sigma}_r,$$

$$\mu_{S,t} = r_t + \sigma_\delta \lambda_\delta + \frac{1}{S_t}\frac{\partial S}{\partial r_t}\sigma_r \lambda_r.$$

By definition, the quantity $-\frac{1}{S_t}\frac{\partial S}{\partial r_t}$, which represents the negative of the sensitivity of the stock price with respect to the short-term rate, is the duration of the stock index. We denote it by $D_{S,t}$. A straightforward differentiation shows that the duration is:

$$D_{S,t} = \frac{\sum_{\substack{i=1 \\ t_i > t}}^{\infty} D(t_i - t)c_{t,t_i}p(t,t_i)}{\sum_{\substack{i=1 \\ t_i > t}}^{\infty} c_{t,t_i}p(t,t_i)},$$

where:

$$c_{t,t_i} = \exp\left[(\mu_\delta - \sigma_\delta \lambda_\delta)(t_i - t) - \frac{\sigma_r \sigma_\delta \rho_{r\delta}}{a}[t_i - t - D(t_i - t)]\right].$$

Thus, duration is a function of time and short-term rate only; in particular it is not impacted by

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dividends. The stock volatility is:

$$\sigma_{S,t} = \sqrt{\sigma_\delta^2 - 2D_{S,t}\sigma_r\sigma_\delta\rho_{r\delta} + D_{S,t}^2\sigma_r^2}.$$

The bond volatility has the familiar expression as the product of duration by interest rate volatility:

$$\sigma_{B,t} = D_{B,t}\sigma_r.$$

We also note that the expression for the volatility vector of the stock bears formal resemblance with the expression for the volatility vector of an inflation-indexed zero-coupon bond (see Martellini and Milhau (2013)). This is not surprising, given that the stock price in our model can be seen as the price of a portfolio of zero-coupon bonds indexed on dividends.

In addition to volatilities and expected returns, the model also implies an explicit expression for the instantaneous stock-bond correlation. This quantity is defined as:

$$\rho_{SB,t} = \frac{\underline{\sigma}_{S,t}'\underline{\sigma}_{B,t}}{\sigma_{S,t}\sigma_{B,t}},$$

so that:

$$\rho_{SB,t} = \frac{D_{B,t}\sigma_r[-\sigma_\delta\rho_{r\delta} + D_{S,t}\sigma_r]}{\sigma_{S,t}\sigma_{B,t}}.$$

Finally, it is also possible to compute the tracking error of the stock with respect to the bond: it is defined as the instantaneous standard deviation of the ratio $\frac{S_t}{B_t}$. An application of Ito's lemma shows that it is equal to the Euclidean norm of the vector $[\underline{\sigma}_{S,t} - \underline{\sigma}_{B,t}]$. Hence:

$$TE_{S,t} = \sqrt{\sigma_{S,t}^2 + \sigma_{B,t}^2 - 2\sigma_{S,t}\sigma_{B,t}\rho_{SB,t}}.$$

All these instantaneous moments (expected returns, volatilities, correlation and tracking error) are stochastic, in that they not only depend on time, but also on the short-term rate; on the other hand, they do not depend on the dividend level in the context of the model.

Numerical Implementation We now study the impact of the dividend process parameters on the tracking error of the stock index with respect to the bond index, as well as the impact on the two main drivers of the tracking error, namely the stock-bond correlation and the stock volatility. As explained in Section 2, the TE is a natural measure of the proximity between a stock and a bond, but it is also interesting to look at the contributions of stock volatility and stock-bond correlation to this measure.

Model-implied volatilities and the correlation are functions of the short-term rate; for a given set of parameter values, we simulate 1,000 paths for the various stochastic processes (interest rate, dividend, stock and bond prices), and we compute the correlation and the stock volatility according to the

previous formulas. We then average the values across dates and states of the world, so as to come up with estimates of the expected correlation, volatility and TE:[11]

$$E[\rho_{SB,t}], \ E[\sigma_{S,t}], \ E[TE_{S,t}].$$

We must specify a base case set of parameter values for the interest rate and the dividend processes. For the interest rate model, we use the same values as Martellini and Milhau (2013), which are calibrated to the US sovereign yield curve over the 1964-2010 period:

$$a = 0.0672, \ b = 0.0360, \ \sigma_r = 0.0231, \ \lambda_r = -0.3385.$$

For the dividend process, we fit a Geometric Brownian motion to the time series of aggregate dividends of US stocks (S&P index) available on Robert Shiller's website [12] for the 1946-2013 period. We obtain:

$$\mu_\delta = 0.0254, \ \sigma_\delta = 0.029.$$

Given it is likely that some stocks will have their dividend process positively correlated to the short-term rate, while the correlation will be negative for some others, it is reasonable to take a neutral view on the correlation $\rho_{r\delta}$ and to set it equal to zero in the base case.

Fixing the price of dividend risk is less straightforward, because this parameter cannot be directly estimated from the data; in particular, it cannot be estimated as the stock Sharpe ratio, since the stock is also exposed to interest rate risk. In order to set it to a realistic level, we search for the value that matches the model-implied Sharpe ratio of the stock averaged across dates and states of the world, and a long-term estimate for a broad US stock index. We use the estimates of Ibbotson (2013) for the long-term excess return and volatility of US large-capitalisation stocks over the 1926-2012 period. These values imply a long-term Sharpe ratio of 0.41. The choice for λ_δ that makes the model replicate this value is $\lambda_\delta = 0.4388$.

We focus on the growth rate and the volatility of the dividend process (μ_δ and σ_δ), and the correlation ($\rho_{r\delta}$) between the short-term rate and the dividend processes. The latter parameter is the one that controls for the "flow-through effect". The general idea is that in the presence of a strongly negative $\rho_{r\delta}$, a positive shock to interest rates increases the discount rate but tends to drive dividends downwards; this unambiguously decreases the stock price, as would be the case for a bond. In this case, we thus expect a positive stock-bond correlation. On the other hand, if $\rho_{r\delta}$ is positive, then the same positive shock will still increase the discount rate, but will tend to increase dividends. This competition between the two effects, with a net impact on the stock price that depends on parameter values, results in a lower bond correlation.

To assess the quantitative magnitude of this effect, we plot in Figure 1 the stock-bond correlation

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[11]It makes sense to average correlation and volatility values across dates, because stock and bond returns in the model are stationary (this would not be the case if the bond had a finite maturity).

[12]http://www.econ.yale.edu/~shiller/data.htm

as a function of σ_δ and $\rho_{r\delta}$. We confirm that this correlation is decreasing in $\rho_{r\delta}$, except of course when dividends are deterministic ($\sigma_\delta = 0$); in this case, the stock is perfectly correlated with the bond, because it behaves like a "fixed- (or at least deterministic-) income" security. Hence, other things being equal, stocks that are most bond-like are those whose dividends are strongly negatively correlated with interest rates. The same mechanics explain why stock volatility is decreasing in $\rho_{r\delta}$; with a large $\rho_{r\delta}$, the fluctuations in the two sources of uncertainty (interest rate and dividends) tend to cancel each other, which decreases stock volatility.[13] On the other hand, a negative $\rho_{r\delta}$ implies that any shock on one of the two state variables will be reinforced by a shock in the opposite direction on the other, which results in higher volatility. Hence, a lower $\rho_{r\delta}$ increases the correlation, which should make the stock more bond-like, but it also increases the volatility, which leads to the opposite conclusion.

Dividend volatility has a more straightforward effect on bond friendliness. A higher σ_δ implies a decrease in the stock-bond correlation, which seems natural because a stock with highly volatile dividends differs substantially from a fixed-income security. In general, it also produces an increase in stock volatility. This is not systematically true, however. We observe in Figure 1 that for large positive correlations, stock volatility is a U-shaped function of σ_δ. This effect arises because the compensation between dividend and interest rate shocks can only take place if dividends are volatile; in other words, a certain degree of uncertainty is needed for the offsetting effect to take place. On the other hand, when dividends become very volatile compared to the interest rate, a marginal increase in their variance translates into higher stock volatility. Apart from this exception, a higher σ_δ makes the stock less bond-like (we shall see in Figure 2 that this result is robust with respect to changes in the value of the dividend drift, μ_δ).

The figure also shows the net effect of σ_δ and $\rho_{r\delta}$ on the TE of stocks with respect to bonds. From the previous observations, it is expected that this tracking error is in general decreasing in σ_δ, since a lower σ_δ implies both a higher correlation and, often, a lower volatility. The only exception to this rule is when $\rho_{r\delta}$ is very high and σ_δ is low; in this case only, an increase in dividend uncertainty may reduce the TE, a property which was observed for stock volatility and which is transmitted to the TE. On the other hand, what we learn from the third graph with respect to the first two ones is how $\rho_{r\delta}$ impacts the TE. For high values of σ_δ, the TE is increasing in $\rho_{r\delta}$, which shows that the volatility effect dominates the correlation effect. This means that stocks with high flow-through effect (i.e. high $\rho_{r\delta}$) are less bond-like than the average. For low values of σ_δ, the TE is a U-shaped function of $\rho_{r\delta}$. This is the result of the conflict between the effects of $\rho_{r\delta}$ on volatility and correlation. Overall, it should be acknowledged that the impact of $\rho_{r\delta}$ is not as strong as that of σ_δ; in other words, reducing dividend uncertainty appears to be a more efficient way of lowering the TE.

In Figure 2, we turn to the joint effects of dividend volatility and growth rate. A first observation

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[13]It can be verified that almost all tested combinations of parameter values imply a stock volatility which is higher than bond volatility: indeed, the average bond volatility implied by our base case parameters, is 15.42% (it is, is of course, the same for all pairs $(\sigma_\delta, \rho_{r,\delta})$), and 88.1% of the tested combinations of parameters lead to a stock volatility above this value.
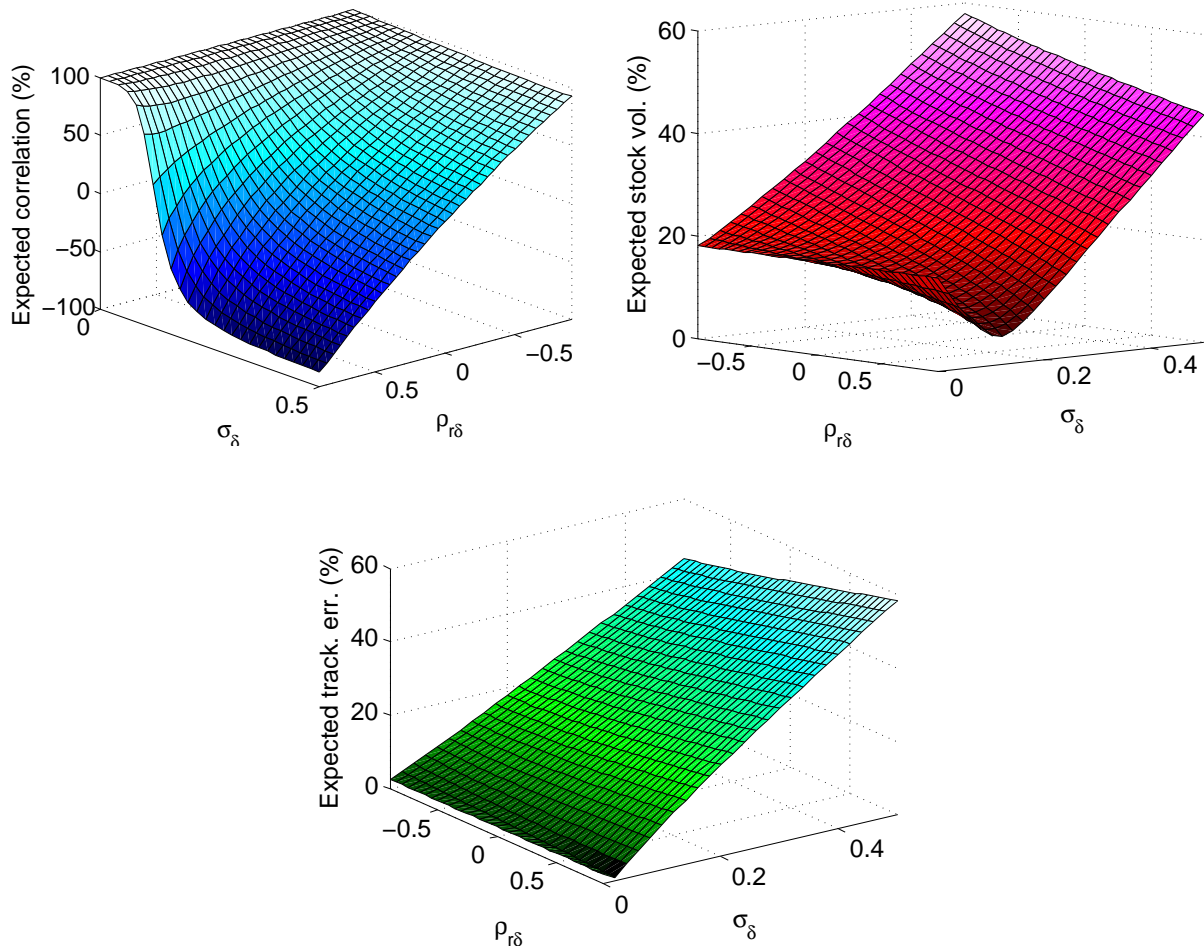
Figure 1: **Impact of dividend volatility and interest-rate-dividend correlation.** This figure shows the model-implied stock-bond correlation, stock volatility and tracking error of stock with respect to bond (these measures are averaged across dates and states of the world), as functions of dividend volatility (σ_δ) and the correlation between the short-term rate and the dividend processes ($\rho_{r\delta}$). Other parameters are fixed at their base case values. Each correlation, volatility and tracking error is computed from 1,000 paths simulated over a 5-year period with a monthly time step (thus, the average is taken over 120×1000 points).

is that σ_δ has the same impact on correlation, volatility and TE for all values of μ_δ: an increase in σ_δ makes the stock less bond-like in all dimensions. The effect of μ_δ is less straightforward. On the one hand, a higher μ_δ increases the correlation with the bond, but it also increases the volatility, and the net effect turns out to be a higher TE. Thus, a higher dividend growth makes the stock less bond-like in the sense of the TE. The impact of μ_δ on volatility can be explained by the impact on duration: a large μ_δ signals high dividend payments in the future, which lengthens stock duration accordingly (future payments become relatively more important than the short-term ones). This longer duration increases stock volatility, as can be seen from the expression of the volatility, recalling that $\rho_{r\delta} = 0$ in the base case.
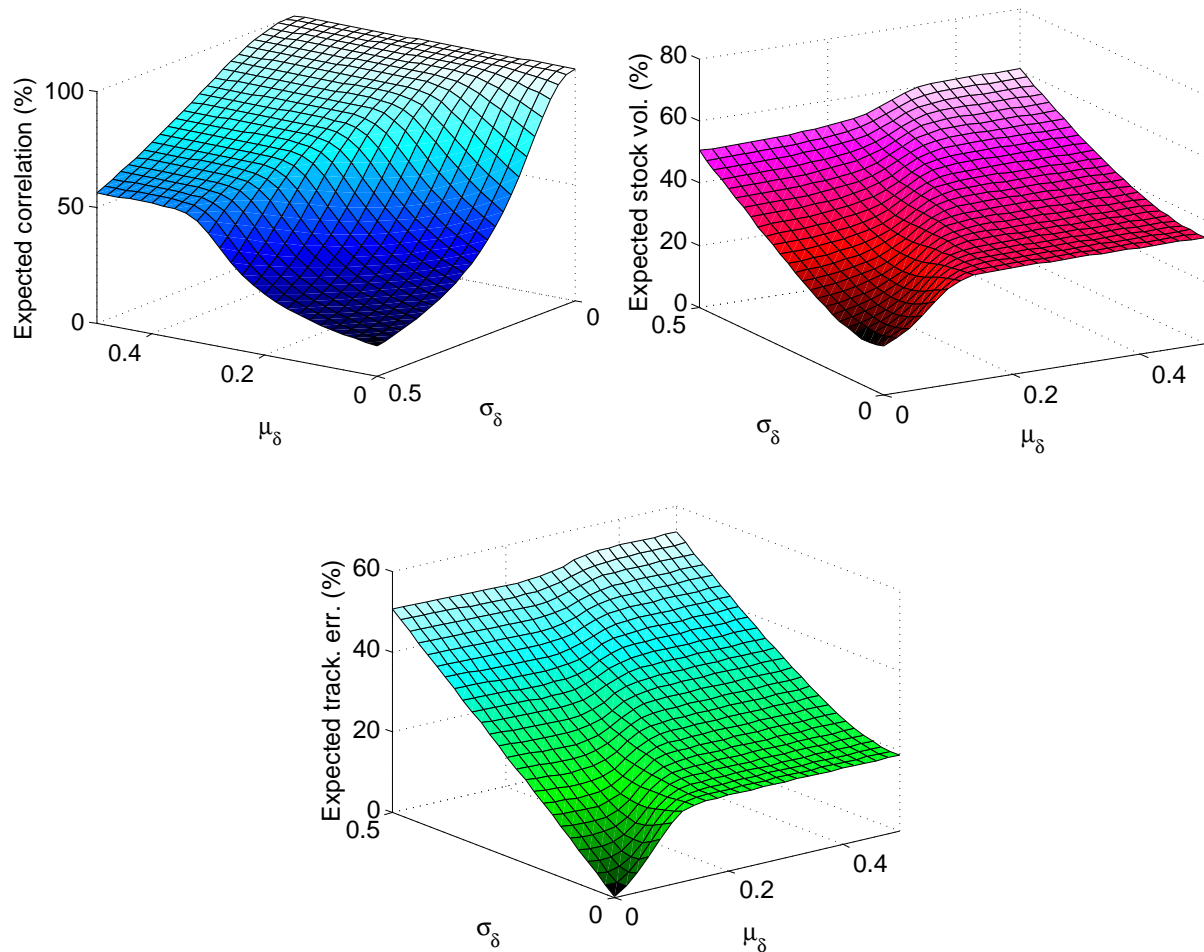
Figure 2: **Impact of dividend volatility and growth rate.** This figure shows the model-implied stock-bond correlation, stock volatility and tracking error of stock with respect to bond (these measures are averaged across dates and states of the world), as functions of dividend volatility (σ_δ) and growth rate (μ_δ). Other parameters are fixed at their base case values. Each correlation is computed from 1,000 paths simulated over a 10-year period with a monthly time step.

In summary, the model highlights the key roles of the parameters σ_δ and $\rho_{r\delta}$ in discriminating bond-like stocks. The impact of σ_δ is clear: other things being equal, stocks that are most bond-like, in the sense of low volatility, high correlation and low TE with bonds, are those with low dividend uncertainty. The impact of the flow-through effect, as measured by $\rho_{r\delta}$, is less straightforward. On the one hand, a lower $\rho_{r\delta}$ increases the stock-bond correlation; on the other hand, it increases stock volatility. As a result, it is not clear whether or not a lower correlation makes the stock more bond-like. The dividend growth has a similar effect. On the one hand, a higher μ_δ partly compensates for dividend uncertainty, which has a positive effect on the correlation of stocks with very volatile dividends. On the other hand, it increases stock volatility, which tends to make stocks less bond-like.

3.3 Summary: Characterisation of Liability-Friendly Stocks

In conclusion, low volatility appears to be a key characteristic of liability-friendly stocks from a factor matching perspective. This can be explained by a first, straightforward, effect. Selecting low volatility stocks mechanically reduces the tracking error with respect to a bond index regarded as a liability proxy as long as stocks are more volatile than bonds. There is also a less obvious effect, which is mentioned in the literature: an analysis in the context of the CAPM suggests that equity duration is decreasing in the market beta. Because low beta is often associated with low volatility, this shows that low volatility stocks are more likely to have positive duration than the high volatility ones. As a matter of fact, the empirical studies conducted in the literature lead to the conclusion that low volatility stocks or defensive sectors have higher bond correlation and higher durations. From a cash-flow matching perspective, low volatility stocks are also attractive. Indeed, both the theoretical model presented in Section 3.2.3 and the empirical analysis of Section 3.1.2 suggest that low volatility of returns is associated with high out-of-sample dividend yield and low dividend yield uncertainty. These multiple favourable features clearly make a case for the selection of low volatility stocks.

The correlation of stock returns with the returns on a bond proxy is also an obvious criterion. The first reason for this is because this correlation measures to what extent stocks are bond-like and it is therefore natural to select the stock with the highest correlation with bonds. The second reason for this is because the tracking error is decreasing in the correlation and hence, stocks with higher correlation will have lower tracking error, all other parameters being equal. The correlation of dividends with interest rates has been shown to be potentially useful in identifying bond-like stocks since it quantifies the flow-through effect. The empirical literature shows that sectors or industries with high pricing power (hence, high flow-through) have low or even negative durations, so the corresponding stocks are not good approximations to bonds, and are correspondingly not well suited for liability matching. In the context of the theoretical model, we find that the correlation has competing effects on stock volatility and stock-bond correlation, the result being a decreasing or a U-shaped relationship between the correlation and the tracking error. Since the effect of this correlation parameter on bond friendliness is not as clear as that of dividend volatility, and in view of the practical difficulties raised by the empirical measurement of the correlation between dividends and interest rates, we do not retain it as a selection criterion.

The direct search for stocks with improved cash-flow matching ability is complicated by the fact that dividend yield volatility is difficult to estimate at the individual stock level. Hence, it is useful to characterise stocks with low dividend yield variability through alternative attributes which are either observable or relatively easy to estimate at the stock level. As mentioned previously, low volatility is one of these characteristics. A high stock-bond correlation may also help in this regard, since the theoretical model shows that such high correlations can only be achieved in the presence of low dividend volatility, unless the high dividend volatility is compensated by a strong flow-through effect (that is, a high correlation between dividends and interest rates). This is yet another justification for

the relevance of the stock-bond correlation criterion. On the other hand, our empirical analysis has suggested that the most effective attribute for discriminating cash-flow matching stocks is the dividend yield itself: high dividend yield stocks do not only imply higher out-of-sample dividend yields, but also lower dividend yield uncertainty. The choice of the dividend yield as a selection criterion is further justified by the results of Baker and Wurgler (2012), who find that a high dividend yield ratio implies a higher bond beta, and by the findings of Leary and Michaely (2011), who show that firms with high dividend yields smooth their dividend more so than their low dividend yield counterparts.

As a conclusion, we have isolated three main criteria which we expect to be discriminating for the sorting of stocks with above average liability-friendliness: low volatility; high correlation with a liability proxy (a bond index); and high dividend yield. We provide a thorough empirical test of the impact of stock selection procedures according to these criteria in the next section, and explore the implications in terms of improvement in investor welfare.

4   Constructing Equity Benchmarks with Improved Liability-Friendliness

We consider a broad investment universe of 500 stocks - those of the S&P 500, extracted from the CRSP/Compustat database - over a period ranging from 1975 to 2012.

4.1   Empirical Methodology

In order to assess liability-friendliness, we will use a proxy which will mimic the returns of a risk-free bond with constant maturity. We proceed as in Section 4.1 of Campbell et al. (2003), using series of US Treasury constant maturity yields.[14] In most of our applications, we take the maturity of the liability proxy to be 15 years, which is the longest maturity for which data is available from 1975 onwards. However, we shall also include robustness checks, in which we will evaluate liability-friendliness with respect to proxies with both shorter and longer maturities. If we denote by $y(t_i, \tau)$ the zero-coupon yield of maturity τ observed at time t_i, the return of a bond with constant maturity τ between dates t_i and $t_{i+1} = t_i + \Delta t$ is computed as

$$r_{t_i, t_{i+1}} = -(D_y(t_i, \tau) - \Delta t)y(t_{i+1}, \tau - \Delta t) + D_y(t_i, \tau)y(t_i, \tau),$$

where we use the approximation $y(t_{i+1}, \tau - \Delta t) = y(t_{i+1}, \tau)$ and $D_y(t_i, \tau) = \frac{1-(1+y(t_i,\tau))^{-\tau}}{1-(1+y(t_i,\tau))^{-1}}$.

At each annual rebalancing date (taken as the Friday that is closest to the 21st of March), we select 100 stocks according to one of the relevant criteria determined in the previous section, that is low volatility, high dividend yield and high correlation with liabilities. Volatility is measured as the standard deviation of weekly returns of the stock over the past two years; correlation is measured as the Pearson correlation of the weekly returns of the stocks with the weekly returns of the liability proxy over the past two years; lastly, dividend yield is computed as the ratio between the total return index

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[14]Available at http://www.federalreserve.gov/pubs/feds/2006/200628/200628abs.html.

divided by the price return index, minus one (so that we have $DY_{t-1,t} = \frac{S_t + D_{t-1,t}}{S_t} - 1 = D_{t-1,t}/S_t$). The total return index (numerator) is computed with the CRSP "Returns" item, while the price return index is computed using the CRSP "Returns without dividends" item.

The portfolio is then composed of an EW combination of these stocks. We will subsequently assess the impact of alternative weighting schemes, including the cap-weighting scheme, on the results. The stocks are kept in the portfolio for one year and after this holding period, new weights are computed and new positions are taken. The weights and returns of the portfolio are collected on a daily basis so as to compute performance indicators over the whole period.

In order to quantify both the liability-friendliness and the performance of the portfolios, we will compute the following indicators.

- Liability-friendliness indicators:

    – The tracking error with respect to the liabilities, defined as the standard deviation of the difference of the daily returns of the portfolio and those of the liabilities, multiplied by $\sqrt{252}$;

    – The annualised volatility of the portfolio, computed as the standard deviation of the daily returns of the portfolio, multiplied by $\sqrt{252}$;

    – The correlation with respect to the liabilities, defined as the Pearson correlation between the daily returns of the portfolio and those of the liabilities;

    – The average annual dividend yield of the portfolio, computed as the average of out-of-sample dividend yields. The time-t out-of-sample dividend yield of a firm is computed as the dividends paid between date $t-1$ and date t, divided by the time-t capitalisation. The dividend yields are then averaged across stocks (depending on their respective weights) and over the 38 years of the sample. This indicator measures the average amount of dividend that was received by a shareholder who invested \$100 in the portfolio at the beginning of each year of the sample and held the portfolio for one year;

- Performance indicators:

    – The annualised total return of the portfolio, computed as

    $$(V_{t_N}/V_{t_1})^{252/N} - 1,$$

    where V is the value of the portfolio, t_N is the last date of the sample and N is the number of trading dates in the sample;

    – The Sharpe ratio of the portfolio, defined as the ratio of the annualised excess return of the portfolio (over the annualised risk-free rate) to the annualised volatility of the portfolio;

– The annual turnover, computed as the sum of changes in positions at rebalancing dates, divided by two:

$$Turnover = \frac{1}{2N} \sum_{n=1}^{N} \sum_{i=1}^{N} |w_{t_n}^i - w_{t_n-}^i|,$$

where $w_{t_n}^i$ is the target weight of the ith asset at rebalancing date t_n, and $w_{t_n-}^i$ is its weight just before the rebalancing.

These indicators are computed over the whole sample period. However, liability-driven investors are particularly interested in the behaviour of their portfolio when interest rates decrease, leading to an increase in liability values. Consequently, we have also computed the annualised returns and correlation with liabilities of the portfolios conditionally on interest rate downturn periods identified in the following way. We compute a 40-day moving average series of the 15-year US treasury constant maturity yield. Based on this smoothed series, we search for periods which decrease for 40 days or more. The annualised return and correlation with liabilities are then calculated over these periods.

4.2 Base Case Results

In line with the conclusions of the previous section, our base case study is focused on three portfolios, each one consisting of an EW combination of one hundred stocks: the high dividend yield, the high correlation and the low volatility portfolios. We compare these selections to the cap-weighted and equally-weighted portfolio of all assets in the universe (the EW and CW benchmark). We expect the selections to be more bond-like than this neutral benchmark and hence to display better indicators in terms of liability-friendliness and cash-flow matching ability. To further confirm the relevance of the selection criteria, we also perform the inverse selections, namely with a focus on low dividend yield, low correlation and high volatility portfolio. If the criteria are drivers of liability-friendliness, then these inverse selections should generate portfolios that are less bond-like than the neutral EW benchmark. Lastly, we also consider random selections of 100 stocks performed at each rebalancing date.

All indicators defined in the previous subsection are presented in Table 2. We start by looking at the liability-friendliness in terms of both factor matching and cash-flow matching ability of the selections, which is respectively assessed through the tracking error with the liability proxy (with correlation and volatility as the two main drivers) and the average dividend yield. In order to ensure that selections significantly improve the neutral benchmark taken to be the EW portfolio without selection (to avoid any biases due to the cap-weighting scheme), we test for the significance of the difference between any given indicator for the benchmark and the corresponding indicator for each portfolio. The details of the tests are provided in the legend of the table. An indicator associated with three stars can be considered as significantly different from that of the benchmark EW portfolio at the 1% level.

	Selections					
	No Sel. (CW)	No Sel. (EW)	Random	High Div Yield	High Correlation	Low Volatility
Panel A: Liability friendliness indicators						
Tracking Error (%)	18.8	19.0	19.4 (*)	17.9 (***)	17.4 (***)	14.6 (***)
Volatility (%)	17.3	17.3	17.7 (*)	16.2 (***)	16.2 (***)	13.0 (***)
Correlation (%)	1.46 (***)	-0.80	-0.85	1.88 (***)	7.58 (***)	7.73 (***)
Cond. Corr. (%)	-0.98	-0.60	-0.54	-1.07	-0.57	-1.18
Avg. Div. Yield (%)	3.18	2.94	2.90	5.85	3.70	4.59
Panel B: Performance indicators						
Ann. Ret. (%)	10.9	13.3	13.0	13.8	14.6	13.2
Sharpe Ratio	0.42 (***)	0.55	0.53	0.62 (*)	0.68 (**)	0.73 (***)
Turnover (%)	4.4	12.2	158.6	23.3	48.9	27.5
Cond. Ann. Ret.(%)	5.5	6.4	7.0	8.8	8.2	9.6

	Low Div Yield	Low Correlation	High Volatility
Panel C: Liability friendliness indicators of opposite selections			
Tracking Error	22.6 (***)	22.1 (***)	27.8 (***)
Volatility (%)	21.0 (***)	20.1 (***)	26.1 (***)
Correlation (%)	-1.8 (**)	-7.5 (***)	-6.7 (***)
Cond. Corr. (%)	-0.34	-1.15	-0.26
Avg. Div. Yield (%)	0.51	2.50	1.38
Panel D: Performance indicators of opposite selections			
Ann. Ret. (%)	11.3	12.4	10.7
Sharpe Ratio	0.36 (**)	0.43 (*)	0.27 (***)
Turnover (%)	27.5	55.1	34.2
Cond. Ann. Ret.(%)	3.3	5.1	1.8

Table 2: **Out-of-sample indicators of selection-based portfolios.** The investment universe is the S&P 500 over the 1975-2012 period (Source: CRSP). We assess the significance of the difference of each indicator with respect to that of the EW benchmark portfolio. The tests for the tracking error and the volatility are performed following the HAC inference methodology detailed in Ledoit and Wolf (2011). The test for the correlation is based on the results of Zou (2007) in the overlapping case. Lastly, the test between the Sharpe ratios stems from the HAC procedure of Ledoit and Wolf (2008). In all tests, the null hypothesis is that the indicator of the portfolio based on selections is equal to that of the equally weighted portfolio of all stocks. The p-values are associated with stars in the following way: zero star if the p-value is greater than 10%, one star (*) if the p-value lies between 5% and 10%, two stars (**) if the p-value is above 1% but below 5% and lastly, three stars (***) if the p-value is smaller than 1%.

We first observe that the random selections generate results which are very close to the EW benchmark and therefore do not improve it on any level. On the other hand, the three liability friendly selections lead to lower tracking errors with respect to the liability proxy, compared to both CW and EW benchmarks with no selection. This lower tracking error stems both from lower volatility

and higher correlations with the liability proxy. Over the sample period for the S&P 500 universe, the EW portfolio of the 20% of stocks with the lowest volatilities has a tracking error of 14.6% with respect to our liability proxy while the EW portfolio of the 20% of stocks with the highest volatilities is almost twice as large at 27.8%. At the same time, the focus on low volatility stocks generates a positive 7.7% correlation with the liability proxy, while the focus on high volatility stocks generates a negative correlation of -6.7%, which confirms that low volatility stocks, which tend to be the low dividend uncertainty stocks, are the ones that tend to be the closest approximations to fixed-income securities.

In fact, the low volatility portfolio is found to have a slightly higher out-of-sample correlation with the liability proxy compared to the high correlation portfolio, as a consequence of the well-documented instability of the stock-bond correlations which implies that past high correlations are not necessarily the best forecasts of future high correlations. In other words, low volatility appears to be a better predictor of future high stock-bond correlation compared to prior high stock-bond correlation. However, there is not much discrepancy in terms of conditional correlation, as they all lie between -1.2% and -0.5%. It should also be noted that the three liability friendly selections also imply higher average dividend yields. The largest figure is associated, as expected, with the high dividend yield selection (5.85%), but the low volatility selection immediately follows (4.59%).

Turning to performance indicators, we find that all selections surpass the CW benchmark in terms of annualised returns and all selections, except the low volatility and random selections, also outperform the EW benchmark (the low volatility selection is outranked by the EW benchmark by 0.1%). Furthermore, we find that the increase in Sharpe ratio compared to the EW benchmark is statistically significant, especially for the low volatility selection (and, to a lesser extent, to the high correlation selection). This finding suggests that the increase of liability-friendliness linked to these criteria is not achieved at the cost of lower risk-adjusted performance, with a corresponding increase in investor welfare which will measured in the next section.

In fact, if we compare the Sharpe ratio of the selections versus the Sharpe ratio of the inverse selections, we find that increased liability-friendliness does not command a risk premium; on the opposite, a positive risk premium is associated to low volatility, high correlation and high dividend yield stocks, with a difference in Sharpe ratios between the upper and lower quintile selections related to these criteria being strongly significant (the associated p-value in the HAC test is much smaller than 0.1%). These findings are consistent with existing results, in particular results on the positive risk premium associated with low volatility stocks (the so-called *low volatility anomaly* of Ang et al. (2006, 2009)) and high dividend yield (see Rozeff (1984) and Chen et al. (1990)). Note that turnover is of course the lowest for the CW benchmark because this portfolio is only affected by changes in the composition of the universe. High dividend yield and low volatility selections generate moderate turnovers ranging between 20% and 30%, which suggests that the time-variation in these attributes is not too strong. On the other hand, the portfolio based on high correlations with the liabilities implies significantly larger turnover, consistent with the fact that such estimated correlations are unstable

over time.

One might wonder whether the competing selection procedures lead to a vastly disjointed subset of stocks, or if the selected stocks tend to be somewhat similar for various selection criteria. To test for this, we define the overlap between two selections by the number of stocks that belong to the two selections, divided by the number of stocks in the selections. To ensure a higher degree of robustness, we carry out this analysis for three different universe sizes, namely $N=50$, 100, 250 (i.e. 10%, 20% and 50% of the total number of stocks). The results are gathered in Table 3.

	Selections			
	High Div Yield	High Correlation	Low Volatility	Random
Panel A: N=50				
High Div Yield	100	24.5	42.1	13.1
High Correlation	24.5	100	26.2	9.3
Low Volatility	42.1	26.2	100	14.4
Random	13.1	9.3	14.4	100
Panel B: N=100				
High Div Yield	100	31.3	46.0	20.4
High Correlation	31.3	100	34.2	16.2
Low Volatility	46.0	34.2	100	22.6
Random	20.4	16.2	22.6	100
Panel C: *N=250*				
High Div Yield	100	53.9	68.0	55.2
High Correlation	53.9	100	59.4	49.4
Low Volatility	68.0	59.4	100	53.5
Random	55.2	49.4	53.5	100

Table 3: **Average overlap between selections.** The investment universe is the S&P 500 over the 1975-2012 period (Source: CRSP). At each rebalancing date, we compute the overlap between two selections as the number of stocks which belongs to the two selections, divided by the number of stocks in the selections. We then average the overlaps over the whole sample.

Comparing Panels A, B and C, we observe that the average overlap is increasing in the number of stocks within the portfolios, which is intuitive because in the limit $N=500$, the overlap can only be 100%. The selections which are the closest in terms of percentage overlap are the dividend yield and the low volatility selections. The high correlation displays lower levels of average overlap and shares on average in the case $N=100$ between 30% and 35% of the stocks of the other selections. In this (base) case, the average overlap is never larger than 50%, which means that the three attributes do lead to different subsets of stocks. Lastly, the overlap of the random selections is close to the percentage of stocks retained (from 10% for 50 stocks to 50 % for 250 stocks), which is a confirmation

of the randomness of the picks.

In an attempt to check for the possible presence of sector biases in the selected portfolios, we finally report in Table 4 the average sector weights for each selection procedure. The first striking observation is that all three liability friendly selections are more tilted towards the Utilities sector, compared to both CW and EW benchmarks. The selections compensate with smaller proportions of Wholesale Trade and Business Equipment stocks. This is consistent with the results of Gupta (2012), who finds that as of 2011, 92% of companies belonging to the Utilities sector are dividend payers in the Dow Jones US 2500 universe. According to Gupta (2012), the sector which has the second highest percentage of dividend payers is the financial sector, which is also overweighted compared to the benchmarks by all selections, except the low volatility one.

Even though the three liability friendly selections share similar patterns, each of them has a specific set of sector exposures. The low volatility selection allocates 40% of the portfolio to Consumer Non-Durables and Utilities, while the high correlation selection assigns 40% to Utilities and Financials. The high dividend yield selections are strongly tilted towards the same sectors, but in the reverse order (nearly 29% for Utilities and 21% for the Finance sector).

	Selections					
	No Sel. (CW)	No Sel. (EW)	Random	High Div Yield	High Correlation	Low Volatility
Consumer Non-Durables	8.5	10.2	10.5	8.3	10.7	15.9
Consumer Durables	5.4	3.8	3.8	3.6	2.2	2.5
Manufacturing	10.5	17.4	17.8	12.0	9.0	11.1
Energy	11.3	5.7	5.7	7.1	3.3	5.4
Chemicals	5.0	4.3	4.3	4.2	3.8	5.0
Business Equipment	13.3	10.6	10.6	1.9	5.0	3.4
Telecommunications	6.3	2.7	2.5	4.3	3.8	3.8
Utilities	4.7	7.1	6.8	28.7	16.5	24.3
Wholesale Trade	7.2	9.4	9.3	3.8	7.5	5.8
Healthcare	8.9	5.6	5.6	1.6	7.2	5.6
Finance	13.4	13.7	13.9	21.1	24.3	13.3
Other	5.6	9.6	9.4	3.5	6.6	3.9

Table 4: **Average sector weights of the selection-based portfolios (%).** The investment universe is the S&P 500 over the 1975-2012 period (Source: CRSP). At each rebalancing date, the aggregate weight of each sector is computed by summing the weights of all stocks within the sector. The weights are then averaged across the rebalancing dates. The sectors are those defined by the 12 industry portfolios on Kenneth French's website (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html).

Overall, we find that all stocks are not born equal in terms of liability-friendliness. We also find that a selection on high dividend yield and/or low volatility levels tend to generate, for a reasonable level

of turnover, a portfolio with significantly improved liability-robustness compared to the CW or EW benchmarks based on the whole universe. A selection based on past correlation with a liability-proxy on the other hand generates more turnover, which can be taken as an indication of some substantial lack of stability on the procedure.

4.3   Robustness Checks

The results of the base case depend on various arbitrary choices (choice of the liability proxy, number of stocks in the selections, weighting scheme, etc.). In this subsection, we test whether our results are robust to changes in these dimensions.

4.3.1   Maturity of Liabilities

We first want to check whether our results one the improvements in out-of-sample tracking error and correlation with the liability proxy are robust with respect to the choice of the duration of the liability proxy. To this end, we repeat in Table 5 the analysis with a 1Y and 30Y constant maturity bond indices. Because 30Y yields are only available from 1985 onwards, we restrict our analysis to the 1985-2012 period in this subsection. Moreover, only the tracking error and correlation with liabilities are impacted by this modification in the protocol, hence we do not provide the other unchanged indicators.

With respect to the tracking error, we again observe that all three liability friendly selections lead to above average liability-friendliness compared to the EW benchmark (however, the random selection adds no value). Among all portfolios, the low volatility selection is the one with the lowest tracking error. These results are consistent across maturities. We also note that the tracking error is increasing in the maturity of the liabilities; this is explained by the fact that the longer the maturity, the more volatile the liability proxy, which everything else equal mechanically implies an increase in tracking error.

The same conclusions hold for the correlations with the liability proxy: the three liability friendly selections improve the benchmark and within the selections, the low volatility ranks first while the high dividend yield ranks last, which confirms the finding of the base case over a longer period. These results are again very favourable to the low volatility selection.

4.3.2   Inflation-Linked Liabilities

We now turn to the situation faced by a pension fund with inflation-linked liabilities. In this case, the returns on the liability proxy are computed as follows:

$$r_{t_i, t_{i+1}} = -(D(t_i, \tau) - \Delta t) z(t_{i+1}, \tau - \Delta t) + D(t_i, \tau) z(t_i, \tau) + \log \left(\frac{\varphi_{t_i + \Delta t}}{\varphi_{t_i}} \right),$$

where the $z(t_i, \tau)$ are the time-t_i zero-coupon yield of Treasury Inflation Protected Securities (TIPS) with maturity τ and φ_t is the value of the Consumer Price Index published by the US Bureau of

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Liab. Maturity	Selections				
	No Selection	High Div Yield	High Correlation	Low Volatility	Random
Panel A: Tracking Error (%)					
1Y	19.2	18.5 (**)	17.8 (***)	14.5 (***)	19.8 (**)
15Y	21.1	20.2 (***)	19.2 (***)	16.2 (***)	21.6 (**)
30Y	24.9	24.0 (**)	23.0(***)	20.4 (***)	25.5 (**)
Panel B: Correlation (%)					
1Y	-12.9	-10.3 (***)	-6.0 (***)	-5.3 (***)	-13.0
15Y	-10.3	-6.4 (***)	-0.7 (***)	-0.2 (***)	-10.4
30Y	-13.7	-9.8 (***)	-5.7 (***)	-4.7 (***)	-13.8

Table 5: **Impact of liability maturity on statistical liability friendliness.** The investment universe is the S&P 500 over the 1985-2012 period (Source: CRSP). For this particular test of robustness, the sample period is 10 years shorter because the data for the 30Y liability proxy is only available from 1985 onwards. Compared to the base case, the indicators are computed with respect to liability proxies with 1 year, 15 years (base case) or 30 years of maturity. The tests for the differences in tracking error and correlation are performed as in the base case with respect to the EW benchmark.

Labor Statistics. In this case, $D_z(t_i, \tau) = \frac{1-(1+z(t_i,\tau))^{-\tau}}{1-(1+z(t_i,\tau))^{-1}}$. The CPI time-series is only available at a monthly frequency, so all returns will be computed on a monthly basis. Moreover, the TIPS data is extracted from the US Federal Reserve website[15] and starts in 1999, which reduced the sample size to 1999-2012. In addition to the 15-year maturity base case, we also consider the smallest and largest maturities for which the data is available over the same period (5-year and 20-year maturities).

As in the preceding subsection, we compute the tracking error and correlation of the portfolios with respect to this new inflation-linked proxy and report the results in Table 6 (we do not provide the other unchanged indicators). The low volatility selection remains the best approach for reducing tracking error with respect to liabilities, followed by the high correlation selection (random selections, if anything, deteriorate liability-friendliness). These conclusions are in line with the results of Table 5 and are not impacted by changes in the maturity of the liabilities. We again acknowledge that the tracking error is an increasing function of the maturity of the liability proxies, essentially because their volatility increases with the maturity. Moreover, the ranges of the values across maturities are smaller in Table 6 because the range in maturities (5 years to 20 years is also smaller - compared to 1 year to 30 years in Table 5).

With respect to the correlation indicator, the improvement is meaningful for the high correlation and low volatility selections (at least +3% in absolute values). The smallest maturity yields the highest correlations, but also the smallest spreads between the portfolios. In this particular setting, the correlation seems to be decreasing with the maturity of the liabilities, but this was not the case in Table 5, so that no further general conclusion can be drawn.

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[15]http://www.federalreserve.gov/pubs/feds/2008/200805/200805abs.html

Liab. Maturity	Selections				
	No Selection	High Div Yield	High Correlation	Low Volatility	Random
Panel A: Tracking Error (%)					
5Y	19.6	19.6	16.2 (***)	12.9 (***)	20.5 (***)
15Y	21.3	21.0	18.0 (***)	14.8 (***)	22.1 (***)
20Y	22.4	22.0 (*)	19.1 (***)	16.0 (***)	23.2 (***)
Panel B: Correlation (%)					
5Y	13.3	10.0 (***)	19.1 (***)	16.6 (***)	13.2
15Y	3.4	6.2 (***)	9.9 (***)	12.3 (***)	4.5 (***)
20Y	1.0	4.1 (***)	7.3 (**)	10.3 (***)	2.1 (**)

Table 6: **Inflation-linked liabilities.** The investment universe is the S&P 500 over the 1999-2012 period (Source: CRSP). For this particular test of robustness, the sample period is only 14 years long because the data for the TIPS yields is only available from 1999 onwards. Compared to the base case, the indicators are computed with respect to liability proxies with 5 years, 15 years (base case) or 20 years of maturity. The tests for the differences in tracking error and correlation are performed as in the base case with respect to the EW benchmark.

4.3.3 Number of Stocks

In our base case protocol, we have set the number of stocks to be held in the portfolio to 100. In Table 7, we extend our results to the cases where we retain either 50 or 250 stocks. The sample period is 1975-2012.

The impact of the number of stocks on the tracking error is hardly noticeable on the dividend yield selection (left column of Panel A). This can be explained by the conflict between two competing effects. On the one hand, reducing the number of stocks generates selections which are more tilted towards a liability-friendly factor (see Panel C, which solely focuses on correlation). On the other hand, as N decreases, there is less room for diversification which results in more volatile portfolios and hence higher tracking error. For the high correlation and low volatility selections, it seems that the first effect dominates the second one because the tracking error in these two cases is indeed increasing in the number of stocks in the portfolio. For the random selection, the second effects overrules the first one. In fact, volatility (and mechanically Sharpe ratio) and tracking error are the only indicator which are impacted by the number of stocks for the random selection. We further note that the lowest tracking error is again achieved by the low volatility selection.

With respect the correlation indicator (Panel C), we confirm that the correlation decreases with the number of stocks. This can only hold when the selection criteria are designed towards liability-friendliness, which is the case for the three portfolios. We observe that for the 50 stock selections, the high correlation selection has a higher out-of-sample correlation compared to the low volatility selection.

Turning to the average dividend yield (Panel D), the impact of the number of stocks is strictly

Nb Stocks	Selections			
	High Div Yield	High Correlation	Low Volatility	Random
Panel A: Tracking Error (%)				
50	18.2 (**)	17.1 (***)	14.0 (***)	19.6 (**)
100	17.9 (***)	17.4 (***)	14.6 (***)	19.4 (*)
250	17.9 (***)	18.0 (***)	16.2 (***)	19.1
Panel B: Volatility (%)				
50	16.7 (**)	16.1 (***)	12.5 (***)	17.9 (**)
100	16.2 (***)	16.2 (***)	13.0 (***)	17.7 (*)
250	16.1 (***)	16.5 (***)	14.6 (***)	17.4
Panel C: Correlation with liability proxy (%)				
50	2.5 (***)	10.7 (***)	10.3 (***)	-0.9
100	1.9 (***)	7.6 (***)	7.7 (***)	-0.9
250	0.8 (***)	3.4 (***)	4.0 (***)	-0.9
Panel D: Average Dividend Yield (%)				
50	6.85	4.04	5.34	2.92
100	5.85	3.70	4.59	2.90
250	4.46	3.23	3.77	2.96
Panel E: Sharpe Ratio				
50	0.58	0.67 (**)	0.72 (***)	0.48 (*)
100	0.62	0.68 (**)	0.73 (***)	0.53
250	0.65 (*)	0.61 (*)	0.67 (**)	0.54
Panel F: Turnover (%)				
50	25.2	55.8	31.7	175.5
100	23.3	48.9	27.5	158.6
250	16.0	33.5	19.2	26.3

Table 7: **Impact of the number of stocks.** The investment universe is the S&P 500 over the 1975-2012 period (Source: CRSP). Compared to the base case, we study the impact of the number of stocks which are selected in the composition of the portfolio. In addition to our initial choice (*N*=100), we test a stricter selection (*N*=50, one tenth of the global universe) and a looser one (*N*=250, one half of all of the stocks). The indicators are computed exactly as in the base case. The tests for the differences in variance, tracking error, correlation and Sharpe ratio are performed with respect to the EW benchmark.

monotonous for the all selections and the dividend yield decreases with *N*. From *N*=50 to *N*=250, the loss in dividend yield is at least equal to 0.75% and can reach 2.4% for the high dividend yield selections.

In light of these results, it is tempting to make a case for selecting a relatively small number of stocks, which implies enhanced liability-friendliness (smaller selections are more pure in their respective tilts) and higher dividend yields. However, as mentioned before, they also leave less room for diversification which may result in poorer performance. The corresponding results in terms of Sharpe ratios and turnover are shown in Panel E and Panel F respectively. We note that there is no clear monotonous impact of the number of asset on the Sharpe ratio. This can again be explained by the combination of two opposed effects: large selections improve the diversification potential, but they

also lead to a lower of exposure to possibly rewarded risk factors. In the case of the high dividend yield selection, the Sharpe ratio is slightly increasing, implying that the first effect should be dominant. For all of the tested selections, the Sharpe ratio of the portfolios consisting in 50 stocks is never the maximum Sharpe ratio suggesting that selecting too few stocks is hardly optimal.

The highest Sharpe ratios are obtained for N=250 for the dividend yield selection and for N=100 for the selections based on statistical criteria. Moreover, in all cases, the turnover is a strictly decreasing function of the number of stocks. Consequently, larger selections are expected to lead to smaller transaction costs. Thus, taking into consideration risk-adjusted performance as well as portfolio rotation does not incite a reduction in the number of stocks from 100 to 50. But increasing N from 100 to 250 deteriorates the liability-friendliness of the selections because it implies higher levels of tracking error. Accordingly, fixing N=100 appears to be a balanced choice between enhanced liability-friendliness and attractive risk-adjusted performance with moderate turnover.

4.3.4   Sample Period

Our sample covers many different market conditions. In order to test the robustness of the results with respect to various sample periods and macroeconomic situations, we split our sample into four periods (three decades and one 8-year window) and perform the selection procedures on these subsamples. We provide the indicators related to liability-friendliness in Table 8, Panel A-D.

The liability friendly selection-based portfolios lead to lower tracking errors and volatilities than the EW benchmark over all periods, apart for the high dividend yield selection in 2005-2012. Apart from the turnover, the random selection is only slightly outperformed by the EW benchmark. Similarly to the base case, the low volatility selection remains the most effective in reducing the tracking error and the volatility over each subsample. With respect to correlations (Panel C), the three portfolios display improvements over the EW benchmark, regardless of the timeframe used to compute the indicator. We note that for some subsamples, the highest correlation is obtained for the high dividend yield selection, which is different from what was obtained in the base case.

With respect to Sharpe ratios, the values are highly unstable within the periods, but the rankings between portfolios are almost constant through time, with the no-selection EW benchmark ranking last apart for one exception and the low volatility stocks ranking first or second but close to the first. The same ranking stability is observed in terms of turnover, with the lowest turnover achieved by the EW benchmark, followed by the high dividend yield selection and the low volatility selection.

We also note that the historical trajectory for dividend yields is overall decreasing over the sample under consideration. The post-1990 levels are roughly equivalent to half of those before 1980. The rankings of the selections are not impacted by the choice of the sample period, with the higher average dividend yield levels obtained by the high dividend yield portfolios followed by the low volatility selections. Among the selection-based portfolios, the high correlation stocks are those which yield the lowest average dividend yields, but they are nonetheless above those of the EW benchmark, which

	Selections				
	No Sel. (EW)	High Div. Yield	High Correlation	Low Volatility	Random
Panel A: Tracking Error (%)					
OVERALL	19.0	17.9	17.4	14.6	19.4
1975-1984	12.8	10.4	12.2	9.8	13.0
1985-1994	14.6	12.9	13.8	12.2	15.0
1995-2004	18.6	16.3	17.5	14.7	19.0
2005-2012	29.2	29.3	25.9	21.4	30.0
Panel B: Volatility (%)					
OVERALL	17.3	16.2	16.2	13.0	17.7
1975-1984	12.8	10.0	12.3	9.1	12.7
1985-1994	15.5	13.9	15.3	13.3	15.3
1995-2004	16.8	14.7	16.4	13.2	17.2
2005-2012	24.1	25.2	21.6	16.9	25.6
Panel C: Correlations with Liabilities (%)					
OVERALL	-0.8	1.9	7.6	7.7	-0.9
1975-1984	31.2	36.2	34.7	36.1	30.7
1985-1994	34.6	38.7	43.4	41.6	34.2
1995-2004	-8.6	-1.9	3.3	1.4	-8.3
2005-2012	-41.7	-35.1	-36.6	-36.1	-40.0
Panel D: Average Dividend Yield (%)					
OVERALL	2.94	5.85	3.7	4.59	2.90
1975-1984	4.70	8.29	5.83	7.06	4.70
1985-1994	3.01	5.91	3.84	4.67	2.91
1995-2004	1.83	4.29	2.52	3.13	1.79
2005-2012	1.92	4.46	2.16	3.00	1.92
Panel E: Sharpe Ratio					
OVERALL	0.55	0.62	0.68	0.73	0.53
1975-1984	1.13	1.65	1.14	1.64	1.12
1985-1994	0.73	0.81	0.77	0.85	0.74
1995-2004	0.70	0.81	1.06	0.85	0.72
2005-2012	0.23	0.22	0.24	0.33	0.19
Panel F: Turnover (%)					
OVERALL	12.2	23.3	48.9	27.0	158.6
1975-1984	11.5	25.0	55.3	27.5	157.3
1985-1994	10.9	23.4	46.2	28.5	160.1
1995-2004	14.2	23.4	44.1	30.5	155.6
2005-2012	11.8	23.6	54.8	24.6	154.9

Table 8: **Robustness over different timeframes.** The investment universe is the S&P 500 over the 1975-2012 period (Source: CRSP). Compared to the base case, we divide the whole sample into four subsamples (three decades and an eight-year period). The indicators are then computed with respect to these subsamples, using the same methodology as in the base case.

confirms that all selections tend to improve the capacity to deliver large dividend streams per dollar invested.

4.3.5 Alternative Selection Criteria

In this subsection, we perform selections based on alternative criteria different from those found to be as economically motivated in previous research (see Section 3). These criteria are the standard observable book-to-market and size attributes, as well as empirical beta and duration measures. We also test the debt-to-equity ratio, in an attempt to see whether there is any relationship between a higher leverage and a higher sensitivity to changes in interest rates, which would lead to a higher liability-friendliness. For the sake of completeness, we look at the two extreme selections, that is we test both the lowest and highest values for the attributes. We also provide the figures of the EW benchmark and those of the extreme selections based on volatility for comparison purposes. The results are gathered in Table 9.

We first observe that among all proposed selections, none outperforms the low volatility stock selection across all indicators of liability-friendliness. The reverse statement holds for high volatility stocks since they have the largest tracking error and volatility and the lowest correlation and average dividend yield with respect to any other selection, re-enforcing the finding that the low volatility selection is the lost relevant discriminating attribute for the purpose of mimicking a bond-like portfolio among all tested selections.

Among the proposed alternatives, the low beta selection has the lowest tracking errors and the highest correlation with liabilities, as well as the highest average dividend yield. High beta stocks display the opposite features. This shows that the intuition conveyed by the CAPM (detailed in Section 3) is verified empirically: low beta stocks are more liability-friendly than high beta stocks. The fact that the book-to-market ratio does not strongly discriminate for liability-friendliness is a confirmation of the findings of Baker and Wurgler (2012).

The leverage ratio is an intuitive criterion because firms which have a higher ratio are expected to be more sensitive to variations is the interest rates.[16] However, we find that even though high leverage stocks are indeed more bond-like than their low leverage counterparts, the differences in liability-friendliness and cash-flow matching capability (first four columns) is much less pronounced than for selections based on beta for instance. The same conclusions hold for selections based on size: large firms are more liability-friendly than small firms, but the differences in liability-friendliness indicators between the two selections is comparable to the differences between selections based on leverage, and not as large as those obtained for selections driven by beta or by volatility. Similarly, our results show that high empirical duration selections lead to portfolios that are more liability-friendly than low empirical duration selections for all indicators (tracking error, correlation with liabilities, average

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[16]Bello et al. (2014) show that high or low levels of leverage can lead to a shift in risk from short horizon to long horizon dividend flows, which can impact the liability-friendliness of stocks.

Selections	Out-of-sample indicators					
	Track. Error (%)	Vol. (%)	Correl. (%)	Avg. DY (%)	Sharpe Ratio	Turn. (%)
No Selection	19.0	17.3	-0.8	2.94	0.55	12.2
Low Volatility	14.6 (***)	13.0 (***)	7.7 (***)	4.59	0.73 (***)	27.5
High Volatility	27.8 (***)	26.1 (***)	-6.7 (***)	1.38	0.27 (***)	39.2
Low Debt/Cap	21.1 (***)	19.4 (***)	-3.2 (**)	2.12	0.47	35.8
High Debt/Cap	20.1 (***)	18.5 (***)	1.1 (*)	3.65	0.48	33.0
Low Cap	21.7 (***)	19.8 (***)	-5.4 (***)	2.46	0.59	30.5
High Cap	19.1	17.6 (*)	2.5 (***)	3.10	0.40 (*)	17.0
Low B/M	19.7 (**)	18.2 (***)	1.4 (*)	2.04	0.26 (***)	32.1
High B/M	20.2 (***)	18.4 (***)	-2.1 (*)	3.42	0.81 (***)	45.8
Low Beta	15.3 (***)	13.6 (***)	5.0 (***)	4.07	0.71 (***)	29.4
High Beta	27.2 (***)	25.6 (***)	-4.6 (***)	2.11	0.27 (***)	35.8
Low Duration	22.5 (***)	20.7 (***)	-5.7 (***)	2.16	0.39 (*)	50.7
High Duration	19.6 (**)	18.1 (***)	3.7 (***)	4.04	0.53	39.4
High SemiDev	26.7(***)	24.9(***)	-6.54 (***)	1.67	0.30 (***)	41.0
Low SemiDev	15.0(***)	13.5(***)	7.36 (***)	4.30	0.72 (***)	35.7
High VaR	27.7(***)	25.9(***)	-7.01 (***)	1.54	0.28 (***)	39.8
Low VaR	14.6(***)	13.1(***)	8.00 (***)	4.41	0.76 (***)	34.4

Table 9: **Alternative selection criteria.** The investment universe is the S&P 500 over the 1975-2012 period (source: CRSP). The Debt/Cap ratio is computed as the sum of Compustat item "Long-Term Debt - Total" plus Compustat item "Short-Term Debt - Total" divided by Compustat item "Capitalization". The latter Compustat item is used to perform the high capitalization selections and the book-to-market sorts as well, the numerator being the Compustat item "Common/Ordinary Equity - Total". The beta is computed as in the CAPM model, where the market factor is the CW benchmark. The regressions are run on weekly returns over 2 years (104 sample points). Moreover, for stability purposes and in order to reduce turnover, we shrink the raw betas towards their sector counterparts, using the method of Vasicek (1973). The durations are evaluated in a similar two-step procedure: their raw value is computed using the regression method of Reilly et al. (2007) with the same data specifications as the beta regressions; then, they are shrunk towards their sector counterparts. The sector factors are the equally-weighted portfolios of all stocks within the sectors. The VaR selection is performed using the 5% quantile of the past 104 weekly returns and the semi deviation is computed as the standard deviation of negative returns among the 104 past returns. We include the low and high volatility selections for comparison purposes. As in Table 2, we display the significance of the difference of indicators with respect to the EW benchmark (first row) using the classical star representation.

dividend yield). However, this criterion does not appear to be as relevant as volatility or beta because the impact of the selection is not as substantial.

In further robustness checks, we have included the Value-at-Risk and the semi-deviation, which are alternative risk measures. The corresponding figures are close to those of the volatility, which can be explained by the fact that the average overlap between the selections is 73% for the semi-deviation and 76% for the VaR. Lastly, in unreported results, we have tested a low tracking error selection. The

figures are hardly distinguishable from those of the low volatility selection, which can be explained by the fact that the correlations are small in absolute value (less than 10%), which leads to an average overlap of 92%.

Overall, these results confirm volatility as the preferred selection criterion. The other base case criteria all perform better than the alternative ones for the indicator that they were intended to optimise: the high correlation displays higher correlations and the high dividend yield selection generates larger average dividend yields, compared to all of the alternatives of Table 9.

Turning to Sharpe ratios, the highest value (0.81) is obtained by the high book-to-market stocks, which is consistent with the well documented risk premium of value stocks (Fama and French (1992)). The low volatility selection ranks second and the high volatility selection ranks second-to-last, thereby yielding the second spread between extreme quintiles of firm attributes (0.46 difference in Sharpe ratio, versus 0.55 difference for selections based on book-to-market).

With respect to turnover, the EW benchmark generates the smallest levels of rotations, followed by the high capitalisation stocks, with size rankings being more stable in the cross-section compared to raking based on other attributes. The largest turnovers stem from duration based selections.

4.3.6 Alternative Weighting Schemes

In results reported so far, all portfolios were equally weighted. Even though the EW benchmark typically enjoys high risk-adjusted performance (see DeMiguel et al. (2009)), one may wonder whether other weighting schemes may improve liability-friendliness, or risk-adjusted performance, or both. To this end, we consider the following four weighting schemes:

1. The capitalisation-weighted (CW) portfolio, where the weight of each stock is proportional to its market capitalisation;

2. The equally-weighted (EW) portfolio (base case), where all stocks have the same weight;

3. The inverse volatility (IV) portfolio, where the weight of each stock is proportional to the inverse of its volatility (computed as the standard deviation of its past weekly returns over the past two years);[17]

4. The minimum variance with weight constraints (MV-C) portfolio: it is computed as the solution

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[17]If we assume that all pairwise correlations are equal, the inverse volatility weighted portfolio is identical to the equal risk contribution portfolio (Maillard et al. (2010)).

of the following optimisation problem:

$$\min_{\underline{w}} \underline{w}'\Sigma\underline{w}$$
$$s.t. \quad \underline{w}'1 = 1$$
$$\frac{1}{3N} \le w_i \le \frac{3}{N},$$

which amounts to minimise the ex-ante variance of the portfolio, subject to the budget constraint (the weights sum to one) and to upper and lower bounds on the weights. The role of these weight constraints is to prevent the optimisation to allocate strictly positive weights to only a very small proportion of the initial universe (as is documented in Clarke et al. (2011)). Because the selection was performed at an earlier stage, we do not want the weighting scheme to further restrict the investable set since it would indeed reduce the diversification potential of the portfolios.

The covariance matrix $\underline{\Sigma}$ is estimated using an implicit factor model with a number of factors determined by a criterion stemming from Random Matrix Theory (see Laloux et al. (2000) for further details).

The results reported in Table 10 show that the IV and MV-C weighting schemes lead to further improvements in terms of tracking error and volatility compared to the EW and CW benchmarks. We observe however, that the improvement in terms of correlation is only true in comparison to the EW portfolio. In fact, for high correlation selections, the highest correlation is reached by the CW benchmark (8.9%). For the other selections, it is attained by the MV-C weighting scheme.

Turning to average dividend yields, the IV and MV-C again outperform their CW and EW counterparts over all selections and the MV-C portfolios rank first in all cases.

This can be explained by the fact that these schemes allocate more weight to stocks with low volatilities, which have, as was shown in the previous section, larger dividend yields on average. The rankings in terms of Sharpe ratio are invariant with respect to the selection criterion: the MV-C rank first, the IV second and the EW third. The high risk-adjusted performance of the MV-C portfolio nevertheless comes at the cost of higher turnover (at least a 10% to 20% increase in absolute value, depending on the selections), compared to other weighting schemes. The EW and IV portfolios have similar levels of turnover. Lastly, we notice that for some indicators, such as volatility, average dividend yield, or Sharpe ratio, the improvement from equal weights to MV-C weights is limited for the low volatility selections, compared to the other selections. This is because the selection step has already tilted the portfolio towards low risk stocks, and therefore, the additional minimum variance layer does not add much value in this case.

The findings of Table 10 can be summarised as follows. The IV and MV-C weighting schemes improve the tracking error, the average dividend yield and the Sharpe ratio. The improvement is more spectacular for the MV-C portfolios, but it comes at the price of higher turnover.

Selections	Weighting Schemes				Selections	Weighting Schemes			
	CW	EW	IV	MV-C		CW	EW	IV	MV-C
Panel A: Tracking error (%)					*Panel D: Average Dividend Yield (%)*				
No Selection	18.8	19.0	17.8 (***)	16.1 (***)	No Selection	3.18	2.94	3.29	3.66
High Div Yield	18.0 (***)	17.9 (***)	16.8 (***)	15.6 (***)	High Div Yield	5.79	5.85	6.01	6.07
High Correlation	17.6 (***)	17.4 (***)	16.8 (***)	15.7 (***)	High Correlation	3.81	3.70	4.08	4.53
Low Volatility	15.6 (***)	14.6 (***)	14.5 (***)	14.1 (***)	Low Volatility	4.24	4.59	4.71	4.80
Random	19.4 (*)	19.4 (*)	19.0 (**)	18.6 (***)	Random	3.09	2.90	3.23	3.61
Panel B: Volatility (%)					*Panel E: Sharpe ratio*				
No Selection	17.3	17.3	16.1 (***)	14.3 (***)	No Selection	0.42 (***)	0.55	0.60 (*)	0.69 (***)
High Div Yield	16.5 (***)	16.2 (***)	15.2 (***)	14.0 (***)	High Div Yield	0.56	0.62 (*)	0.65 (**)	0.70 (***)
High Correlation	16.4 (***)	16.2 (***)	15.5 (***)	14.4 (***)	High Correlation	0.58	0.68 (**)	0.69 (**)	0.73 (***)
Low Volatility	14.2 (***)	13.0 (***)	12.9 (***)	12.5 (***)	Low Volatility	0.60 (*)	0.73 (***)	0.73 (***)	0.74 (***)
Random	17.8 (**)	17.7 (**)	17.3	16.9 (**)	Random	0.47 (*)	0.53	0.55	0.60 (*)
Panel C: Correlation (%)					*Panel F: Turnover (%)*				
No Selection	1.5 (***)	-0.8	1.1 (***)	3.5 (***)	No Selection	4.4	12.2	13.0	30.8
High Div Yield	3.2 (***)	1.9 (***)	3.8 (***)	5.5 (***)	High Div Yield	26.0	23.3	22.8	43.5
High Correlation	8.9 (***)	7.6 (***)	8.5 (***)	8.6 (***)	High Correlation	60.5	48.9	50.6	71.5
Low Volatility	7.9 (***)	7.7 (***)	8.3 (***)	9.3 (***)	Low Volatility	31.9	27.5	27.8	51.9
Random	-0.2 (*)	-0.9	-0.3	-0.5	Random	166.7	158.6	160.2	167.2

Table 10: **Added value of weighting schemes.** The investment universe is the S&P 500 over the 1975-2012 period (Source: CRSP). The procedure is the same as in the base case. At each rebalancing date, in addition to the equally-weighted average of all stocks, we consider the weighting schemes defined in Section 4.3.6 (namely, CW, IV, MV-C). These weighting schemes are applied after the selection step.

4.3.7 Double Sorts

Given that low volatility selections appears to dominate in terms of factor matching properties, while high dividend yield selection appear to dominate in terms of cash-flow matching properties, it is

tempting to test whether a combination of these two selection criteria may lead to an even better outcome on both dimensions. For the sake of completeness we also test a selection criterion based on correlation and consider a large number of possible ways to combine the criteria. As there is theoretical and empirical evidence that volatility is a strong driver of liability-friendliness, we will perform sorts which all involve the volatility attribute. This is also motivated by the fact that selecting low risk stocks does not deteriorate the performance of the portfolios. In order to remain coherent with the base case framework, we aim to select 100 stocks. There are many arbitrary ways to proceed, but we focus on two of them: either we start by selecting, or we start by eliminating. If we start by selecting, then we retain only 200 stocks in the first selection, but if we start by eliminating, we retain 400 stocks. In the first case, we tilt the portfolio towards the criterion, but in the second, we disqualify stocks which have an abnormally low (or high) characteristic, while keeping a sufficiently large universe for the second selection criterion to be more decisive. The results are collected in Table 11.

Selections	Out-of-sample indicators						
	Tracking Error (%)	Volatility (%)	Correlation (%)	Avg. DY (%)	Annualised Return (%)	Sharpe ratio	Turnover (%)
No Sel. (EW)	19.0	17.3	-0.8	2.94	13.3	0.55	12.2
VOL200-DY100	15.2 (***)	13.7 (***)	6.6 (***)	5.45	13.7	0.73 (***)	23.5
VOL400-DY100	16.7(***)	15.1 (***)	3.3 (***)	5.83	13.8	0.67 (**)	22.8
VOL200-COR100	15.2 (***)	13.9 (***)	9.6 (***)	4.24	13.8	0.72 (***)	41.7
VOL400-COR100	16.5 (***)	15.2 (***)	8.8 (***)	3.86	14.6	0.72 (***)	47.5
COR200-VOL100	15.4 (***)	14.0 (***)	9.0 (***)	4.22	13.5	0.70 (***)	41.8
COR400-VOL100	14.6 (***)	13.2 (***)	8.5 (***)	4.53	13.3	0.73 (***)	31.5
DY200-VOL100	15.0 (***)	13.5 (***)	6.9 (***)	5.25	13.6	0.74 (***)	24.9
DY400-VOL100	14.6 (***)	13.1 (***)	7.8 (***)	4.66	13.0	0.71 (***)	27.0

Table 11: **Double sorts.** The investment universe is the S&P 500 over the 1975-2012 period (Source: CRSP). At each rebalancing date, we proceed to a double sorting of the stocks. For instance, of the first row of the table, we start by selecting the 200 stock which have the lowest volatility. Then, among these 200 stocks, we pick those which have the highest dividend yield. The abbreviations stand for: VOL=low volatility, DY=high dividend yield, COR=high correlation with the liabilities.

From the numbers in Table 11, it is hard to find a particular portfolio which outperforms the others across most indicators. For instance, the four selections which involve the correlation criterion do have larger out-of-sample correlations with the liabilities, but their volatilities, tracking errors and average dividend yield are not as competitive. Likewise, the first two selections of the table have the largest average dividend yields, but those are mitigated by higher volatilities and tracking errors, as well as lower correlations with liabilities.

The lowest volatilities (13.1% and 13.2%) are reached by the two selections which start by eliminating (i.e. retain 400 stocks) and then sorting according to volatility. This allows for the volatility criterion to be decisive, which is not the case for the other double sorts. This observation does not

hold for the correlation indicator because as noted earlier on, the volatility attribute is as good a predictor of future correlation as the correlation attribute. Consequently, the highest correlations are obtained when combining the correlation and the volatility criteria. Likewise, the highest average dividend yields stem from selections involving the dividend yield and the volatility. We however note that the pure dividend yield selection reached a level of 5.85% of divided yield and is therefore not outperformed by double sorts. The Sharpe ratios are not significantly different (testing the highest minus the lowest yields a HAC p-value above 15%) as they range between 0.67 and 0.74. Lastly, the reported turnovers confirm those from Panel F of Table 10: selections based on correlation imply higher asset rotations, while the lowest ones are obtained by combining volatility and dividend yield criteria.

Overall, the best selections are those which have balanced performances and combine competitive liability-friendliness, increased cash-flow matching abilities and possibly high risk-adjusted performance. Among the eight portfolios proposed in this subsection, the DY200-VOL100 displays such qualities. Its volatility and tracking error are low and its average dividend yield and annual return are among the highest. As such, it is among the most bond-like portfolios of all we have tested while keeping reasonably high levels of returns. This is in fact not surprising if we recall the decomposition of the variance of total returns:

$$V(R_{t,t+1}) = V\left(\frac{S_{t+1}}{S_t}\right) + V\left(\frac{D_{t,t+1}}{S_t}\right) + 2Cov\left(\frac{S_{t+1}}{S_t}, \frac{D_{t,t+1}}{S_t}\right).$$

When we compute the realised volatility, we use weekly returns and dividend cash-flows are blended in these returns, which means that the focus is essentially set on the first term (which makes sense because it is by far the largest one). The first dividend yield selection allows us to first consider the second term (high dividend yield portfolios are associated with lower variance in dividend growth) by eliminating the stocks with low cash-flow matching ability. Therefore, combining the features is a way to reduce the variance of total returns in two steps by focusing on different terms of the decomposition of the total variance. It is also a method that provides both enhanced liability-friendliness (low tracking error) and increased cash-flow matching capability (high average dividend yield) while maintaining a competitive risk-adjusted performance.

Consequently, the DY200-VOL100 selection appears as an attractive equity benchmark in the context of an LDI strategy. Moreover, if we combine this selection with the minimum-variance weighting scheme with box constraints (MV-C), then the liability-friendliness is further improved, and the indicators reach the following attractive levels: 14.1% of tracking error, 12.5% of volatility, 8.2% of correlation and 5.4% of average dividend yield. Overall, double sorts starting with DY and then low volatility generate comparable levels of factor matching liability-friendliness with improved cash-flow matching (average DY) properties compared to selection purely based on volatility. The Sharpe ratio increases to 0.79, but the turnover also rises to 46.0%. This is another illustration of how diversification through portfolio optimisation can further improve both the liability-friendliness and the risk-adjusted

performance of a portfolio. Moreover, if we look at the tracking error with respect to inflation-linked liabilities (15-year maturities), it decreases to 14.6%, which is significantly lower than the 21.3% of the EW benchmark. Accordingly, this particular portfolio seems very attractive in an asset-liability management context.

4.4 Assessing the Benefits of Liability-Friendly Equity Benchmarks in LDI Strategies

We have proposed a methodology suited for building equity portfolios which exhibit a higher degree of liability-friendliness compared to standard stock indices, and we now would like to measure the resulting improvement in investor welfare. As explained in the introduction, the intuition is that liability-driven investors may be able to invest a higher portion of their portfolios in equities when using a more liability-friendly benchmark for the *same* ALM risk budget, which can be measured in terms of funding ratio volatility, which a risk-averse liability-driven investor would want to minimise. If the equity benchmark with improved liability-friendliness also happens to dominate the CW index in terms of performance, then the investor will enjoy the joint benefit of having more dollars invested in equity for the same risk budget, as well as a better reward earned for each dollar invested. On the other hand, if the equity benchmark with improved liability-friendliness is dominated by the CW index in terms of performance, then the trade-off between higher liability-hedging benefits and lower performance will result in a net welfare gain or loss depending on parameter values.

All welfare gains will be reported with respect to the CW S&P 500 index, which is the default choice for most investors. Among all of the possible alternative equity benchmarks discussed so far, we focus on the following five strategies: (i) the constrained minimum variance portfolio of all stocks; (ii) the EW portfolio of the low volatility stocks; (iii) the constrained minimum variance portfolio of low volatility stocks; (iv) the EW portfolio of the previously discussed DY200-VOL100 double sort; and (v) the constrained minimum variance portfolio of the DY200-VOL100 selection. These five choices can be categorised depending on the sources of improvement in liability-friendliness. For (i), the improvement comes from the weighting scheme because no selection is performed. For (ii) and (iv), the enhancement stems from the selection, but not from the weighting scheme, which is not designed to improve liability-friendliness, even though it may contribute to superior risk-adjusted performance. Lastly, for (iii) and (v), both the selection and the weighting scheme will contribute to improving the liability-friendliness. Risk and performance indicators for the six portfolios under consideration in this section are summarised in Table 12.

We note that all proposed portfolios significantly improve the CW benchmark over all indicators except of course turnover. In this context, we expect that a switch from the S&P 500 index to an improved benchmark will unambiguously lead to welfare gains for a liability-driven investor. Focusing first on improvements in risk-adjusted performance, and excluding the MV-C portfolio of all assets, for which no selection has been performed, we find that all Sharpe ratios lie between 0.73 and 0.79, versus

Portfolios	Out-of-sample indicators						
	Tracking Error (%)	Volatility (%)	Corr. (%)	Avg. DY (%)	Ann. Return(%)	Sharpe Ratio	Turnover (%)
All / CW (S&P 500)	18.8	17.3	1.5	3.18	10.9	0.42	4.4
All / MV-C	16.1 (***)	14.3 (***)	2.4 (*)	3.66	13.5	0.69 (***)	30.8
Low Vol/ EW	14.6 (***)	13.0 (***)	7.7 (***)	4.59	13.2	0.73 (***)	27.5
Low Vol / MV-C	14.1(***)	12.5 (***)	8.0 (***)	4.80	13.0	0.74 (***)	51.9
DY200-VOL100 / EW	15.0 (***)	13.5 (***)	6.9 (***)	5.25	13.6	0.74 (***)	24.9
DY200-VOL100 / MV-C	14.1 (***)	12.5 (***)	8.2 (***)	5.40	13.6	0.79 (***)	46.0

Table 12: **CW versus liability-friendly portfolios.** The investment universe is the S&P 500 over the 1975-2012 period (Source: CRSP). The procedure is the same as in the base case. In the names of the portfolios, the selection is separated from the weighting scheme by a slash. The tests for the tracking error and the volatility are performed following the HAC inference methodology detailed in Ledoit and Wolf (2011) . The test for the correlation is based on the results of in Zou (2007) the overlapping case. Lastly, the test between the Sharpe ratios stems from the HAC procedure of Ledoit and Wolf (2008). In all tests, the null hypothesis is that the indicator of the alternative portfolio is equal to that of the S&P500. The p-values are represented using the usual star representation.

0.42 for the S&P 500 index. Regarding enhancement in hedging benefits, we also find that substantial improvements are obtained, regardless of the indicator used to measure liability-friendliness (tracking error, volatility, correlation, or average dividend yield).

Overall, the MV-C of all assets appears to be the least liability-friendly amongst the alternative equity benchmarks since it leads to the highest tracking error and volatility, and the lowest correlation and average dividend yield, a result that confirm the usefulness of the selection step. On the other hand, the DY200-VOL100/MV-C portfolio, which appears as the most liability-friendly portfolio, combines the benefits of both relevant stock selection and weighting procedures.

We now turn to the analysis of indifference curves (Figure 3), which represent in the (return of the funding ratio, tracking error) parameter space the set of parameter values for which the investor welfare remains constant.

Compared to the CW benchmark, all proposed portfolios achieve a positive improvement in LUG leading to a minimum 1% welfare gain. The smallest gain (1.45%) is obtained by the MV-C portfolio of all stocks and the largest gain (1.88%) is achieved with the double sort procedure associated with the MV-C weighting scheme. This portfolio has the same tracking error as the low-volatility MV-C portfolio but a higher annualised return. Conversely, it has approximately the same annualised return as the MV-C portfolio of all stocks (up to a 0.1% difference), but a much smaller tracking error (2% difference in absolute value).

To provide an easier way to interpret the measure of welfare gain, we now compute the gain in expected log funding ratios of the aggregate LDI strategy, defined in Proposition 2.4. Because the expression involves the unobservable (and investor-dependent) risk-aversion parameter γ, we adjust
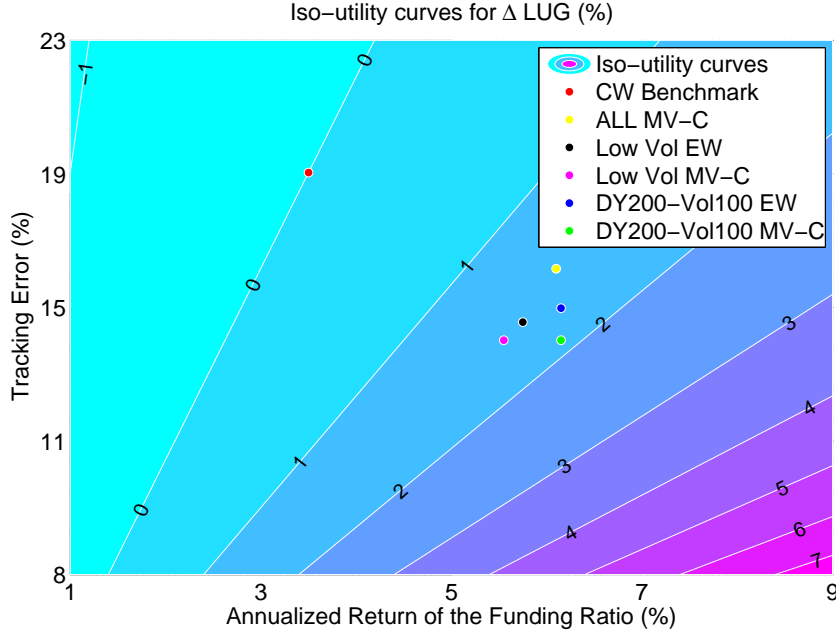
Figure 3: **Indifference curves.** This figure shows the increase in LUG as a function of the tracking error and the annualized return of the funding ratio as defined in Proposition 2.5. The allocation in the CW benchmark is equal to 40%. The white lines indicate the points where the increase is exactly equal to $\pm 1\%$, $\pm 2\%$, etc. The reference portfolio is the CW benchmark (which by definition has a 0% ΔLUG). The portfolios with increased liability-friendliness are represented as small coloured circles.

the allocation to equities in the case of the improved benchmark consider so as generate the exact same funding ratio volatility as when investing a given fraction of the portfolio to equities when using S&P 500 index as a benchmark, as explained in Section 2. Because the S&P 500 index has a tracking error with respect to the liability proxy which is higher than the tracking error achieved with the selected portfolios, investors may allocate a higher proportion of their portfolios to equities in order to match the variance of the funding ratio when the equity benchmark is improved (the proportion is given by the ratio of tracking errors, as discussed in Section 2). We report the results in Figure 4, where we observe that the steepest slope is associated to the double sort combined with the MV-C weighting scheme. The MV-C portfolio of all stocks leads to the smallest increase in funding ratio expected return because it is penalised by its comparatively larger tracking error, which is at least 1% higher in absolute value compared to the other improved portfolios. Overall, we find that for a 40% equity allocation, the gain in annual excess return reaches a minimum of 1.45% compared to the S&P 500 benchmark (the crossing points at the 40% level correspond to the dots in Figure 4).

In Figure 5, we also plot the historical evolution of the funding ratio over the sample period under the assumption of a 40% allocation to the S&P 500 equity benchmark as a default option, with the corresponding risk and performance indicators for the funding ratio reported in Table 13.

In Figure (5a), we note that all improved liability-friendly portfolios outperform the S&P 500 over
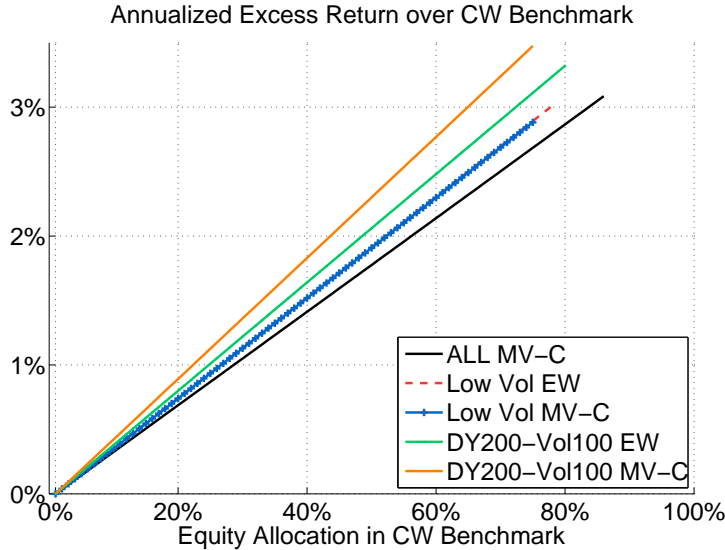
Figure 4: **Increase in LUG compared to the CW S&P 500 equity benchmark.** This figure shows the increase in LUG between an LDI strategy which takes the CW benchmark as equity building block and the strategies which take the selection-based portfolios as building blocks. The latter strategies are constructed so that the variance of the funding ratio is equal in all cases to that of the base case (i.e. when the equity block is the CW benchmark and the equity allocation is equal to 40%). The lines stop before 100% of equity allocation because reaching such levels would imply an equity allocation for the selection-based portfolios of more than 100% in order to match the variance of the funding ratio.

the sample period based on the same 40% equity allocation. The numbers in Table 13 show that for a 40% equity allocation, the volatility of the funding ratios is between 1.1% and 1.9% lower in absolute value when the equity benchmark is the liability-friendly portfolio. With the exception of the MV-C portfolio of all stocks, the reduction of the maximum drawdown reaches at least 10% in absolute value, or 30% in relative value.

In Figure (5b), we observe that the outperformance is more spectacular when the allocation to the improved equity benchmark is adjusted (increased) so as to generate the same volatility of the funding ratio as when investing 40% in the S&P 500 index. The resulting increase in equity allocation combined with the improved performance of the alternative benchmarks leads to even higher outperformance with respect to the S&P 500 index.

Overall, we find that differences in performance between LDI strategies based on S&P 500 equity benchmark versus those based on improved equity benchmarks are substantial. When the allocation to the improved benchmark is chosen so as to match the funding ratio volatility measured with a 40% allocation to the S&P 500 index, leading for example to a 53.3% equity allocation with the DY200-VOL100/MV-C strategy, the LDI strategies based on the improved benchmark generate on average an excess performance that ranges between 1.2% and 1.6% annually. Interestingly, while funding ratio volatility has been set to comparable levels for both standard and improved benchmarks, the use of
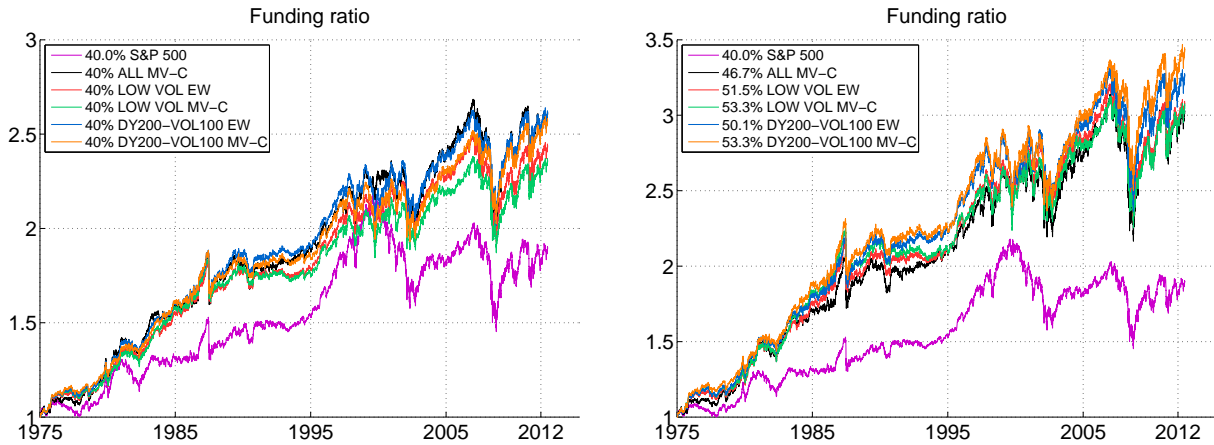
Figure 5: **Historical trajectories for the funding ratio.** These figures show the historical trajectory of the funding ratio from 1975 to 2012. The time series of the funding ratio is computed as the ratio between the values of the global portfolios (equity and bonds) divided by the values of the liability proxy. On the left figure (a), the LDI strategy consists of allocating 40% to the equity block and 60% to the bond block. On the right figure (b), the equity allocations were computed so that the variances of all funding ratios were equal to that of the 40% allocation in the S&P500 (the target volatility for the funding ratios is 7.54%).

the improved benchmark leads to a lower maximum funding ratio drawdown compared to the use of the S&P 500 index. For example, the strategy based on an MV-C portfolio based on the double-sort selection has a maximum drawdown which is almost 8% lower compared to the strategy based on its S&P 500 index counterpart. This suggests that if an investor defines an ALM risk budget in terms of extreme risk (measured by the maximum drawdown of the funding ratio returns) rather than average risk (defined in terms of funding ratio volatility), then an even higher allocation to the benchmark with improved-liability hedging properties can be obtained for the same risk budget, which in turn would lead to even more substantial outperformance levels.

It is further possible to disentangle the contribution to the improved performance which comes from an increased allocation to the equity building block and the contribution which is generated by a higher reward per dollar invested in equities. This exercise is carried out in Table 14 when we switch from a 40% allocation to the CW benchmark to an iso-volatility 53.3% allocation in the DY200-VOL100-MVC portfolio. The resulting increase in equity allocation for the same ALM risk budget, combined with an improved risk-adjusted performance of the dedicated equity benchmark with respect to the S&P 500 index, leads to an improvement in performance reaching 158 basis points annualised over the 1975-2012 sample period. This improvement can be decomposed into a contribution purely emanating from the increase in equity allocation assuming no impact on performance (39 basis points) and a contribution purely emanating from the improved performance of the equity benchmark assuming no increase in allocation (119 basis points).

In the spirit of subsection 4.3.2, we now investigate whether the welfare improvements are robust with respect to the introduction of inflation-indexation in the liabilities. In this case, the liabilities'

Portfolios	(a) 40% Equity Allocation			(b) Iso-vol Allocation		
	Vol. (%)	Max DD (%)	Av. Ret. (%)	Vol. (%)	Max DD (%)	Av. Ret. (%)
All / CW (S&P500)	7.54	33.4	1.73	7.54	33.4	1.73
All / MV-C	6.44	27.1	2.58	7.52	31.0	2.97
Low Vol / EW	5.84	23.0	2.39	7.53	28.9	3.02
Low Vol / MV-C	5.64	21.5	2.30	7.52	27.9	3.00
DY200-VOL100 / EW	6.02	23.3	2.58	7.54	28.6	3.18
DY200-VOL100 / MV-C	5.66	19.7	2.54	7.54	25.7	3.31

Table 13: **Risk indicators for the funding ratio.** The investment universe is the S&P 500 over the 1975-2012 period (Source: CRSP). The procedure is the same as in the base case. In the names of the portfolios, the selection is separated from the weighting scheme by a slash. The time series of the funding ratio is computed as the ratio between the values of the global portfolios (equity and bonds) divided by the values of the liability proxy. On the left figure (a), the LDI strategy consists of allocating 40% to the equity block and 60% to the bond block. On the right figure (b), the equity allocations were computed so that the variances of all funding ratios were equal to that of the 40% allocation in the S&P500 (the target variance for the funding ratios is 7.54%). The volatility is computed as the standard deviation of fund ratio log-returns multiplied by $\sqrt{252}$ and the average return is computed as the mean of fund ratio log-returns multiplied by 252.

returns are proxied by the returns on the 15-year TIPS yields and the Consumer Price Index log-returns, as described in subsection 4.3.2 In Figure 6, we compare the LDI strategy based on the DY200-VOL100 MV-C portfolio to that strategy that relies on the S&P 500. We consider the 40% equity allocation (in blue and green) and the iso-variance allocation for the liability-friendly portfolio (in red).

The most striking result again is the risk reduction implied by the liability-friendly-driven strategies. For an identical 40% equity allocation, the volatility is 23% smaller and the maximum drawdown is 45% lower (in relative values) when switching from the S&P 500 benchmark to a liability-friendly benchmark. The corresponding improvements in average returns are also impressive, reaching 270=303-33 basis points when equity allocation is adjusted so as to generate the same funding ratio volatility.

These results confirm that when the liabilities are inflation-linked, a liability-driven investor will strongly benefit from the use of an equity benchmark that has been designed to have improved liability-friendliness, even if liability-friendliness was measured with respect to a nominal bond proxy. This result can be explained by the fact that interest rate risk is the dominant short-term risk factor in inflation-linked liabilities (see Martellini and Milhau (2013) for more details).

	Equity Allocation						
Panel A: Original equity allocation	10%	20%	30%	40%	50%	60%	70%
Equity benchmark = CW (1)	0.49	0.94	1.35	1.73	2.07	2.38	2.65
Equity benchmark = Liab. friendly	0.66	1.31	1.93	2.54	3.12	3.69	4.23
Panel B: Enhanced equity allocation	13.3%	26.7%	40.0%	53.3%	66.7%	80.0%	93.3%
Equity benchmark = CW	0.64	1.22	1.73	2.18	2.56	2.88	3.14
Equity benchmark = Liab. friendly (2)	0.88	1.73	2.54	3.31	4.05	4.75	5.42
Panel C: Excess return (2)-(1)	0.40	0.79	1.19	1.58	1.98	2.37	2.77
Excess return from increased allocation	0.10	0.19	0.29	0.39	0.49	0.58	0.68
Excess return from increased performance	0.30	0.60	0.90	1.19	1.49	1.79	2.09

Table 14: **CW versus liability-friendly portfolio.** The investment universe is the S&P 500 over the 1975-2012 period (Source: CRSP). We report the average log-returns of the funding ratios, in %, for various allocations in either the CW benchmark, or the DY200-VOL100-MVC liability friendly portfolio. The contributions to the excess return reported in Panel C are simply computed using the simple decomposition formula: $x_{LF}r_{LF} - x_{CW}r_{CW} = (x_{LF} - x_{CW})r_{CW} + (r_{LF} - r_{CW})x_{LF}$.

5   Conclusions and Extensions

While the *fund separation theorem* advocates a clear split between performance and hedging, the *fund interaction theorem* suggests that, everything else equal, investor welfare is increased when the PSP displays attractive liability-hedging benefits. In order to design PSPs with enhanced hedging properties, and empirically measure the resulting impact on investor welfare, we identify two main attributes, high dividend yield and low volatility, leading to the selection of stocks with above average liability-friendliness. Our empirical results show that selecting 100 stocks in the S&P 500 universe according to these criteria indeed leads to statistically significant out-of-sample improvements in portfolio liability-friendliness measured according to various indicators. In particular, we find that the out-of-sample tracking error of the equally-weighted portfolio decreases from 27.8% for a high volatility selection down to 14.6% for a low volatility selection over the 1975-2012 sample period, while the average dividend yield rises from 0.51% for the low dividend yield selection to nearly 6% for the high dividend yield selection.

This evidence of high levels of cross-sectional differences in out-of-sample measures of liability-friendliness is robust with respect to changes in the duration of the liabilities and the presence of inflation indexation. Moreover, we show that combining the two criteria within a double-sort procedure leads to further improvements in terms of each measure of liability-friendliness. Hence, joint selections based on volatility and dividend yield reach a 15% tracking error and 5.3% in average dividend yield. Interestingly, the increase in liability-friendliness is not obtained at the cost of lower risk-adjusted performance. In fact, Sharpe ratios of liability-friendly portfolios are systematically above those of equally-weighted and the cap-weighted benchmarks. Our analysis also suggests that security selection is not the only mechanism leading to enhanced liability-friendliness; the use of suitable weighting
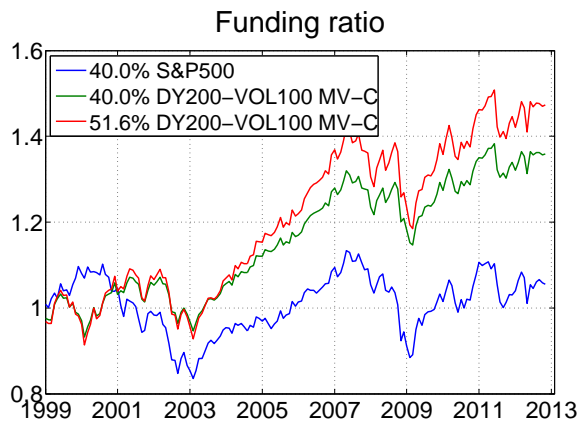
Figure 6: **Inflation-linked liabilities.** The investment universe is the S&P 500 over the 1999-2012 period (Source: CRSP). The procedure is the same as in the base case, except that liabilities are linked with inflation (see subsection 4.3.2 for further details). The volatility is computed as the standard deviation of fund ratio log-returns multiplied by $\sqrt{12}$ (monthly returns) and the average return is computed as the mean of fund ratio log-returns multiplied by 12.

schemes, such as constrained minimum variance weighting schemes, leads to further improvements in both liability-hedging properties and risk-adjusted performance of the selected portfolios, even though minimum variance optimisation alone, without the benefits of a preliminary selection of stocks with above-average liability-hedging benefits, only leads to modest improvements in liability-friendliness.

In the end, the combination of a double sort on volatility and dividend yield and a constrained minimum variance optimisation leads to a portfolio that reaches substantially lower levels of tracking error with respect to a liability proxy (14% on the sample period, compared to 18.8% for the S&P 500 index) while leading to a significantly higher Sharpe ratio (0.79, versus 0.42 for the S&P 500 index) and high average dividend yield levels (5.4% on average, versus 3.18% for the S&P 500 index). As a result of the improvement in liability-hedging benefits, a liability-driven investor allocating for example 40% to equities on the basis of the S&P 500 index can allocate as much as 53.3% to this improved equity benchmark for the same funding ratio volatility, and for a maximum drawdown relative to the liabilities going down from 33.4% to 25.7%. The resulting increase in equity allocation for the same (or lower) risk budget leads to an improvement of performance with respect to the use of the CW S&P 500 benchmark reaching close to 160 basis points (annualised for nominal liabilities over the 1975-2012 sample period). In the case of inflation-linked liabilities over the shorter 1999-2012 sample period, the improvement in performance reaches 270 basis points.

Our analysis can be extended in many directions, notably including a focus on other factors impacting not only the liability side, but also the asset side of investors' balance sheets. In particular, we may seek to assess whether investors could capture some portion of a risk premium supposed to be earned by investing outside equity markets by selecting stocks with above average correlation with the corresponding risk factor. One example would be the design of a dedicated equity benchmark portfolio

that could be used to harvest some part of the credit risk premium on behalf of large investors who might fear that they would not be able to find in corporate bond markets the required amount of liquidity needed.

A Proof of Proposition 2.2

For clarity, we use in the remainder of this appendix the notations A_T and F_T for the terminal wealth and funding ratio achieved with an initial capital A_0. If we write $\frac{dA_t}{A_t} = [r_t + \mu_A]dt + \underline{\sigma}'_A d\underline{z}_t$ and $\frac{dL_t}{L_t} = [r_t + \mu_L]dt + \underline{\sigma}'_L d\underline{z}_t$, then applying Ito's lemma twice, we obtain the dynamics for the funding ratio $F = \frac{A}{L}$:

$$\frac{dF_t}{F_t} = \frac{dA_t}{A_t} - \frac{dL_t}{L_t} - \frac{d\langle A_t, L_t \rangle}{A_t L_t} + \frac{d\langle L_t \rangle}{L_t^2},$$

which implies:

$$\frac{dF_t}{F_t} = [\mu_A - \mu_L - \underline{\sigma}'_A \underline{\sigma}_L + \sigma_L^2]dt + [\underline{\sigma}_A - \underline{\sigma}_L]' d\underline{z}_t.$$

Hence, the drift and the volatility of the funding ratio are constant, so that the funding ratio on date T is log-normally distributed. Specifically, the integration of the previous stochastic differential equation gives:

$$\ln F_T = \ln F_0 + \left[\mu_A - \mu_L + \frac{1}{2}(\sigma_L^2 - \sigma_A^2) \right] T + [\underline{\sigma}_A - \underline{\sigma}_L]' \underline{z}_T,$$

so that:

$$E[\ln F_T] = \ln F_0 + \left[\mu_A - \mu_L + \frac{1}{2}(\sigma_L^2 - \sigma_A^2) \right] T,$$

and

$$V[\ln F_T] = \|\underline{\sigma}_A - \underline{\sigma}_L\|^2 T,$$

$\| \cdot \|$ denoting the Euclidian norm. The expected utility from the terminal funding ratio is given by the textbook expression for the expectation of a log-normal distribution:

$$E[U(F_T)] = \frac{1}{1-\gamma} \exp\left[(1-\gamma)E[\ln F_T] + \frac{(1-\gamma)^2}{2} V[\ln F_T] \right]$$

$$= \frac{1}{1-\gamma} F_0^{1-\gamma} \exp\left[(1-\gamma)\left[\mu_A - \mu_L + \frac{1}{2}(\sigma_L^2 - \sigma_A^2) \right] T + \frac{(1-\gamma)^2}{2} \|\underline{\sigma}_A - \underline{\sigma}_L\|^2 T \right].$$

With an investment in cash only, the terminal funding ratio is F_T^c. The expression for the expected utility is formally equivalent to the one derived for the LDI strategy, but the expected excess return and the volatility are zero, so that:

$$E[U(F_T^c)] = \frac{1}{1-\gamma} F_0^{1-\gamma} \exp\left[(1-\gamma)\left[-\mu_L + \frac{1}{2}\sigma_L^2 \right] T + \frac{(1-\gamma)^2}{2} \sigma_L^2 T \right].$$

Starting from the initial capital yA_0, as opposed to A_0, one multiplies this expected utility by the factor $y^{1-\gamma}$. The condition of equality between the expected utilities from the LDI portfolio and from the portfolio invested in cash implies that y must satisfy:

$$y^{1-\gamma} = \frac{E[U(F_T)]}{E[U(F_T^c)]},$$

By definition, the LUG is the logarithm of y, hence:

$$LUG = E[\ln F_T] + \frac{1-\gamma}{2} V[\ln F_T] - E[\ln F_T^c] - \frac{1-\gamma}{2} V[\ln F_T^c].$$

The LUG is thus a difference of two quadratic utilities. For the optimal strategy, we have:

$$\mu_A = \frac{1}{\gamma} \underline{\mu}_t' \Sigma_t^{-1} \underline{\mu}_t + \left(1 - \frac{1}{\gamma}\right) \underline{\sigma}_L' \sigma_t \Sigma_t^{-1} \underline{\mu}_t,$$

$$\sigma_A^2 = \frac{1}{\gamma^2} \underline{\mu}_t' \Sigma_t^{-1} \underline{\mu}_t + \frac{2}{\gamma}\left(1 - \frac{1}{\gamma}\right) \underline{\sigma}_L' \sigma_t \Sigma_t^{-1} \underline{\mu}_t + \left(1 - \frac{1}{\gamma}\right)^2 \underline{\sigma}_L' \sigma_t \Sigma_t^{-1} \sigma_t' \underline{\sigma}_L,$$

$$\|\underline{\sigma}_A - \underline{\sigma}_L\|^2 = \frac{1}{\gamma^2} \underline{\mu}_t' \Sigma_t^{-1} \underline{\mu}_t - \frac{2}{\gamma^2} \underline{\sigma}_L' \sigma_t \Sigma_t^{-1} \underline{\mu}_t + \left(\frac{1}{\gamma^2} - 1\right) \underline{\sigma}_L' \sigma_t \Sigma_t^{-1} \sigma_t' \underline{\sigma}_L + \underline{\sigma}_L' \underline{\sigma}_L.$$

The terms which depend only on the liabilities will vanish when taking the difference with the terms related to F_T^c. After further computations, we get

$$LUG = \frac{1}{2\gamma} \underline{\mu}_t' \Sigma_t^{-1} \underline{\mu}_t + \left(1 - \frac{1}{\gamma}\right) \underline{\sigma}_L' \sigma_t \Sigma_t^{-1} \underline{\mu}_t + \frac{(1-\gamma)^2}{2\gamma} \underline{\sigma}_L' \sigma_t \Sigma_t^{-1} \sigma_t' \underline{\sigma}_L,$$

where

$$\underline{\mu}_t' \Sigma_t^{-1} \underline{\mu}_t = \underline{\Lambda}' \underline{V}_t' \Sigma_t^{-1} \underline{V}_t \underline{\Lambda} = \underline{\Lambda}' \Omega^{-1} \underline{\Lambda} = \lambda_{PSP}^2,$$

$$\underline{\sigma}_L' \sigma_t \Sigma_t^{-1} \sigma_t' \underline{\sigma}_L = \underline{\sigma}_L' \underline{U} \Omega^{-1} \underline{U}' \underline{\sigma}_L = \sigma_L^2 \underline{R}_L' \Omega^{-1} \underline{R}_L = \sigma_L^2 \rho_{LHP,L}^2,$$

and

$$\underline{\sigma}_L' \sigma_t \Sigma_t^{-1} \underline{\mu}_t = \underline{\sigma}_L' \underline{U} \Omega^{-1} \underline{\Lambda} = \sigma_L \underline{R}_L' \Omega^{-1} \underline{\Lambda}' = \sigma_L \lambda_{PSP} \rho_{PSP,L}.$$

B   Proof of Proposition 2.4

The setting of the proof is identical to the previous one. With two assets only (the equity benchmark S and the bond benchmark B), the dynamic budget constraint takes the simpler form:

$$\frac{dA_t}{A_t} = x\frac{dS_t}{S_t} + (1-x)\frac{dB_t}{B_t}.$$

Substituting the expressions for the dynamics of S and B, we obtain:

$$\frac{dA_t}{A_t} = [r_t + x\mu_S + (1-x)\mu_B]dt + [x\underline{\sigma}_S + (1-x)\underline{\sigma}_B]'d\underline{z}_t.$$

In what follows, we therefore let $\mu_A = x\mu_S + (1-x)\mu_B$ and $\underline{\sigma}_A = x\underline{\sigma}_S + (1-x)\underline{\sigma}_B$ denote the expected excess returns and the volatility vector of the wealth process.

To finish the derivation of the expressions of Proposition 2.4, we need to write the moments of the log funding ratio as functions of those of the two building blocks. We have, for the LDI strategy:

$$\sigma_A^2 = \|\underline{\sigma}_A\|^2 = x^2\sigma_S^2 + (1-x)^2\sigma_B^2 + 2x(1-x)\sigma_S\sigma_B\rho_{S,B},$$

hence,

$$E[\ln F_T] = \ln F_0 + [x\mu_S + (1-x)\mu_B]T + [-\mu_L + \frac{\sigma_L^2}{2}]T - \frac{1}{2}[x^2\sigma_S^2 + (1-x)^2\sigma_B^2 + 2x(1-x)\sigma_S\sigma_B\rho_{S,B}]T,$$

$$V[\ln F_T] = [x^2\sigma_S^2 + (1-x)^2\sigma_B^2 + 2x(1-x)\sigma_S\sigma_B\rho_{S,B}]T - [\sigma_L^2 - 2x\sigma_S\sigma_L\rho_{S,L} - 2(1-x)\sigma_B\sigma_L\rho_{B,L}]T.$$

C   Proof of Proposition 2.5

Let A_T^0 and F_T^0 be the terminal wealth and the funding ratio achieved with Strategy 0. We take a similar definition for Strategy 1 (A_T^1 and F_T^1). For $i = 0, 1$, the variance of the log funding ratio can be written as:

$$V[\ln F_T^i] = V[\ln A_T^i] - [\sigma_L^2 - 2x_i\sigma_{Si}\sigma_L\rho_{Si,L} - 2(1-x_i)\sigma_B\sigma_L\rho_{B,L}]T.$$

The variance-matching condition states that $V[\ln F_T^1] = V[\ln F_T^0]$, which implies that:

$$V[\ln A_T^1] - V[\ln A_T^0] = 2[x_1(\sigma_{S1}\sigma_L\rho_{S1,L} - \sigma_B\sigma_L\rho_{B,L}) - x_0(\sigma_{S0}\sigma_L\rho_{S0,L} - \sigma_B\sigma_L\rho_{B,L})]T.$$

Moreover, the expected log funding ratio is, for $i = 0, 1$:

$$E[\ln F_T^i] = \ln F_0 + [x_i\mu_{Si} + (1-x_i)\mu_B]T + \left[-\mu_L + \frac{\sigma_L^2}{2}\right]T - \frac{1}{2}V[\ln A_T^i],$$

hence the change in LUG is given by:

$$\Delta LUG = E[\ln F_T^1] - E[\ln F_T^0]$$
$$= x_1(\mu_{S1} - \mu_B)T - x_0(\mu_{S0} - \mu_B)T - \frac{1}{2}[V[\ln A_T^1] - V[\ln A_T^0]].$$

This can be rewritten as:

$$\Delta LUG = x_1(\mu_{S1} - \sigma_{S1}\sigma_L\rho_{S1,L} - \mu_B + \sigma_B\sigma_L\rho_{B,L})T - x_0(\mu_{S0} - \sigma_{S0}\sigma_L\rho_{S0,L} - \mu_B + \sigma_B\sigma_L\rho_{B,L})T.$$

To relate the content of the right-hand side to the expected returns and volatilities of the ratios $\frac{S_0}{L}$ and $\frac{S_1}{L}$, we write the dynamics of these ratios. Ito's lemma gives, for $i = 0, 1$:

$$\frac{d(\frac{S_{it}}{L_t})}{\frac{S_{it}}{L_t}} = [\mu_{Si} - \mu_L - \sigma_{Si}\sigma_L\rho_{Si,L} + \sigma_L^2]dt + [\underline{\sigma}_{Si} - \underline{\sigma}_L]'d\underline{z}_t.$$

Hence, the expected return and volatility (tracking error) of $\frac{S_i}{L}$ are given by:

$$\mu_{Si/L} = \mu_{Si} - \mu_L - \sigma_{Si}\sigma_L\rho_{Si,L} + \sigma_L^2,$$

$$TE_{Si} = \sigma_{Si}^2 - 2\sigma_{Si}\sigma_L\rho_{Si,L} + \sigma_L^2.$$

Thus, the variation in LUG admits the following expression:

$$\Delta LUG = x_1\mu_{S1/L}T - x_0\mu_{S0/L}T.$$

Since the variance-matching allocation to the benchmark S1 satisfies $x_1 TE_{S1} = x_0 TE_{S0}$, we have:

$$\Delta LUG = \left[\frac{TE_{S0}}{TE_{S1}}\mu_{S1/L} - \mu_{S0/L}\right]x_0 T.$$

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